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Towards structural controllability of local-world networks

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ABSTRACT

Controlling complex networks is of vital importance in science and engineering. Meanwhile, local-world effect is an important ingredient which should be taken into consideration in the complete description of real-world complex systems. In this letter, structural controllability of a class of local-world networks is investigated. Through extensive numerical simulations, firstly, effects of local world size M and network size N on structural controllability are examined. For local-world networks with sparse topological configuration, compared to network size, local-world size can induce stronger influence on controllability, however, for dense networks, controllability is greatly affected by network size and local-world effect can be neglected. Secondly, relationships between controllability and topological properties are analyzed. Lastly, the robustness of local-world networks under targeted attacks regarding structural controllability is discussed. These results can help to deepen the understanding of structural complexity and connectivity patterns of complex systems.

1. Introduction

The last two decades have witnessed the rapid development of complex networks in capturing different topological patterns of real-world complex systems, ranging from the communication networks (Internet, World Wide Web), biological systems (protein interaction networks, neutral networks), to traffic networks and social networks [1–7]. The significance of connectivity patterns of complex networks to their collective behaviors and functional performance has been verified by fruitful research outcomes so far [8,9]. Especially, since the ultimate proof of our understanding of natural or technological systems is reflected in our ability to control them, controlling a complex network to any desired state is one of the most challenging problems in science and engineering, which has aroused a wide attention from different aspects, including pinning control [10,11], structural controllability [12,13], exact controllability [14], etc.

The concept of structural controllability, firstly proposed by Lin in Ref. [12], was extended to complex networks recently by Liu et al. [13]. According to minimum inputs theorem, they successfully implemented the maximum matching algorithm to estimate the minimum number of controllers and their locations to ensure the structural controllability of complex networks [15]. Stimulated by their work, a lot of efforts have been devoted on structural role of complex networks on controllability as well as its extensions such as control centrality [16,17], control profile [18], control energy [19], controllability optimization [20,21], and so on.

Structural controllability of a network can also be greatly affected by other important topological properties and dynamics occurring on networks. Clustering coefficient and the community structure are found to have no systematic effect while the symmetries of the underlying matching problem can produce linear, quadratic or no dependence on degree correlation coefficients, depending on the nature of the underlying correlations [22]. Additionally, nodal dynamics may come as the focal issue to determine structural controllability of a network [23]. Moreover, Menichetti et al. [24] pointed out that the density of low in-degree and out-degree nodes of a network determines the controllability. "Robustness and fragility", an important property of complex networks, has also been combined with the extensive studies of structural controllability [25–28]. Lately, the study on controllability has also been extended to multilayer networks [29].

Recently, a novel network model proposed by Li and Chen [30], which is called local-world evolving network model, has received a great deal of attention. In order to deeply understand the evolving mechanism of real-world complex systems, it is of great concern to take local-world effect, which is an important feature of many real-world systems, into account. By adjusting an important model parameter – local world size M, generated networks show a transition between exponential and scale-free networks with respect to the degree distribution $p(k)$, which represents the probability

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that a randomly selected node in a network has k connections. After their work, massive research efforts are focused on local-world evolving networks. For example, the local-world effect has been extended to the construction of various network models, such as weighted networks [31–33], bipartite networks [34] and in particular hypernetworks [35]. Furthermore, other evolving mechanisms are also incorporated, including clustering [36], community [37,38], hierarchical structures [39], etc. Moreover, since localization is an essential evolution characteristic of many real-world networks, many variants of local-world network models are proposed and employed to analyze the structure and function of real systems, such as world trade web [40], power grid [41], energy supply-demand network [42] and so on. In addition, such local-world effect leads to various characteristics and collective behaviors on evolving networks, including error and attack tolerance [43], synchronizability [44], epidemics [45], cascading failure caused by load redistribution [46], consensus of multi-agent systems [47], etc.

As mentioned above, structural controllability of local-world networks deserves sufficient discussion, which, however, has not been explored. In this letter, we first introduce as preliminary knowledge the local-world evolving network model, followed by a brief overview of basic concepts of structural controllability. Here, the structural controllability can be quantified through the number of driver nodes which is only determined by the connection topology of the network. Through extensive numerical simulations, effects of local-world size M and network size N on structural controllability are firstly examined, followed by the analysis of relations between controllability and other topological properties. Lastly, structural controllability against random and targeted attacks is investigated.

2. Models

2.1. Local-world evolving network model

The local-world evolving model [30] inherits two ingredients from Barabási–Albert (BA) model [7]: growth and preferential attachment. However, the difference exists in the preferential attachment mechanism: it involves local preferential attachment to capture the localization effect during the evolution of real networks. The iterative algorithm of this model is outlined as follows:

- **Growth:** Starting from a small number m_0 of isolated nodes, at each time-step t, add a new node with m ($m \leq m_0$) edges connecting to the network.
- **Local preferential attachment:** Before connecting the new node to m existing nodes, randomly select M nodes referred to as the "local world"; then, add edges between the new node and m nodes in the local world, the linking probability between any node i in the local world and the new node is:

$$\Pi_{local}(i) = (\frac{M}{m_0+t})(\frac{k_i}{\sum_{j\in local} k_j}). \quad (1)$$

After T time steps, the algorithm results in a network with $N = m_0 + T$ nodes and $E = mT$ edges. In BA model, preferential linking probability is $\Pi(i) = k_i/\sum_j k_j$ and the summation is valued over the whole network. While the linking probability is valued only within the local world of the new node (see Eq. (1)). Two special (limiting) cases exist. If $M = m$, the preferential attachment mechanism does not take effect and the model results in a network with a degree distribution following an exponential form: $p(k) \sim e^{-k/m}$. On the other hand, if $M \geq m_0 + T - 1$, the model reduces to a BA model with $p(k) \sim k^{-\gamma}$ and $\gamma = 3$. By varying the parameter M, network generated by this model represents transitional behaviors between these two extremes.

2.2. Structural controllability of complex networks

Considering a static network (i.e., the edges and nodes are fixed without evolving/switching) associated with a linear time invariant (LTI) system,

$$\dot{x} = A'x(t) + Bu, \quad (2)$$

where $A' \in R^{N\times N}$ denotes the transpose of the network adjacency matrix A, and N is the number of nodes in the network. As for $A = \{a_{ij}\}$, if there is an edge from node i to node j, $a_{ij} = 1$; otherwise $a_{ij} = 0$. $x(t) = [x_1(t), x_2(t), \ldots, x_N(t)]' \in R^N$ denotes the state of a system of N nodes at time t. $B \in R^{N\times M}$ denotes the input matrix which identifies the nodes controlled by an outside controller and M is the number of controllers on the network. For $\forall b_{ij} \in B$, $b_{ij} \neq 0$ if there is a controller j placed on node i, or $b_{ij} = 0$. $u(t) = [u_1(t), u_2(t), \ldots, u_M(t)]' \in R^M$ denotes the input signals from the external controllers on the network. Based on Kalman's state controllability condition, the LTI system (2) is said to be state controllable if and only if the controllability matrix

$$[(A')^{N-1}B, (A')^{N-2}B, \ldots, (A')B, B] \quad (3)$$

has full rank.

The maximum matching algorithm [15] is implemented to estimate the minimum number of controllers and their locations to ensure the structural controllability of complex networks according to minimum inputs theorem [13]. The minimum number of inputs to fully control a network, denoted as N_I, is equal to the minimum number of driver nodes, denoted as N_D. $N_D = 1$ if there is a perfect matching in a network, and any node can be the driven node. Otherwise, N_D is equal to the number of unmatched nodes to a maximum matching and these unmatched nodes are driven nodes. That is,

$$N_I = N_D = max(N - |m^*|, 1), \quad (4)$$

where $|m^*|$ denotes the number of matched nodes to a maximum matching.

Realizing that N_D depends only on network topology, the impact of a particular topology on the network controllability can be represented by a single quantity n_D, denoted to be

$$n_D = N_D/N. \quad (5)$$

Obviously, $0 < n_D \leq 1.0$. A higher value of n_D implies that more driver nodes are needed in order to fully control the whole system. Thus, the smaller the value n_D, the easier the networked system can be controlled, and *vice versa*.

3. Structural controllability of local-world networks

In this section, we focus on the relationship between the structural controllability and the topology of local-world networks.

3.1. Effects of local world size M and network size N

Having reduced the problem of controllability to examining the value n_D, we now consider the impacts of local world size M and network size N on structural controllability of local-world networks.

For clarity, we take $m_0 = m$ in the construction of the local-world evolving networks. Fig. 1 shows the dependencies of n_D on local world size M with different m. All the networks are with size $N = 2000$. And each data point is the average of 10 independent runs.

For sparse networks, i.e., $m = 1$ or $m = 2$, for smaller M, as M increases, n_D is observed to increase. However, n_D becomes