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Experimental demonstration of quantum contextuality on an NMR qutrit

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ABSTRACT

We experimentally test quantum contextuality of a single qutrit using NMR. The contextuality inequalities based on nine observables developed by Kurzynski et al. are first reformulated in terms of traceless observables which can be measured in an NMR experiment. These inequalities reveal the contextuality of almost all single-qutrit states. We demonstrate the violation of the inequality on four different initial states of a spin-1 deuterium nucleus oriented in a liquid crystal matrix, and follow the violation as the states evolve in time. We also describe and experimentally perform a single-shot test of contextuality for a subclass of qutrit states whose density matrix is diagonal in the energy basis.

1. Introduction

The existence of contextuality in the quantum description of physical reality is a fundamental departure from any classical theory. For classical systems, a joint probability distribution exists for the results of any set of joint measurements on the system, and the results of a measurement of a variable do not depend on the measurements of other compatible variables. In non-contextual hidden variable theories, one can hence pre-assign the values to measurement outcomes of observables before the measurement is actually performed [1]. Quantum mechanics on the other hand, precludes such a description of physical reality. In quantum mechanics there exists a context among the measurement outcomes, which forbids us from arriving at joint probability distributions of more than two observables. Given observables A, B and C, such that A commutes with both B and C, while $[B, C] \neq 0$; a measurement of A along with B and a measurement of A along with C, may lead to different measurement outcomes for A. Thus, to be able to make quantum mechanical predictions about the outcome of a measurement, the context of the measurement needs to be specified.

The first test for contextuality was proposed by Kochen and Specker [2], wherein they used a set of 117 rays to reveal the contextuality of a single qutrit (the KS theorem). A modified KS scheme based on 33 rays was constructed by Peres [3]. Since then, there have been various proposals to reduce the number of observables required to demonstrate quantum contextuality. For state-dependent contextuality tests, only a subset of quantum states show contextuality while other measurements can be explained by deterministic non-contextual hidden variable models [4]. State-independent contextuality tests can be represented by an orthogonality graph [5]. A state-independent test of quantum contextuality on a qutrit using a set of thirteen projectors was developed [6] and was later proved to be optimal [7]. Kurzynski et al. [8] showed that in the case of a qutrit, a contextuality test based on a set of nine measurements is able to reveal the contextuality of all single-qutrit states except the maximally mixed state (which saturates their constructed inequality). More recent studies have focused on testing the contextuality of indistinguishable particles and mixed qutrit states [9,10].

Experimental implementations of contextuality tests have been performed by different groups using photonic qutrits [11–13]. KS inequalities on qubits have been tested experimentally using a solid-state ensemble NMR quantum computer [14]. Hardy-like quantum contextuality has been experimentally observed by performing sequential measurements on photons [15]. Furthermore, the connection of contextuality with computational speedup via magic state distillation has been explored [16], and the use of a single qutrit as a quantum computational resource has also been experimentally demonstrated [17,18].

In the current paper, we experimentally demonstrate the contextuality of a qutrit using NMR. On the basis of a set of nine measurements, an experimental test for contextuality is designed and implemented on an NMR qutrit. This involves recasting the original Kurzynski inequality [8] in terms of traceless observables, which can be measured in an NMR experiment. The Gell-Mann matrices provide a natural set to be used in this new scheme,

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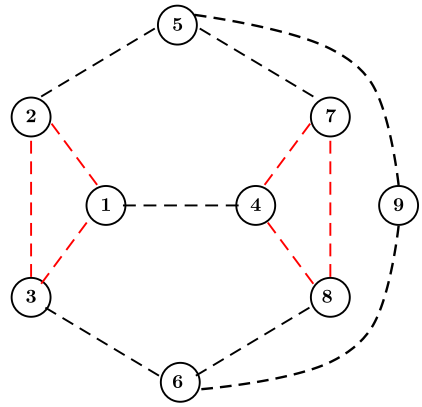

Fig. 1. An orthogonality graph G with the nine projectors as the vertices and the edges $E(G)$ denoting the orthogonality relations between different vertices. An edge between two vertices indicates that the corresponding projectors are orthogonal.

as they are traceless and hence measurable by NMR. We use a deuterium nucleus (spin-1) oriented in a liquid crystalline environment as the NMR qutrit, with the effective quadrupole moment of the spin contributing to two non-overlapping resonances in the NMR spectrum at thermal equilibrium.

The material in this paper is arranged as follows: Section 2 describes the single-qutrit state-independent contextuality inequality and the reformulation of this inequality in terms of an experimentally feasible set of measurements in NMR. Section 3 presents the experimental implementation of single-qutrit contextuality, and Section 4 contains a few concluding remarks.

2. State-independent test with nine observables

The contextuality test for a single qutrit proposed by Kurzynski et al. [8] consists of nine measurements that can reveal the contextuality of all single-qutrit states (other than the maximally mixed state represented by the identity density operator). The set of nine projectors are represented as

$$\Pi_i = |\psi_i\rangle\langle\psi_i|, \quad i = 1, 2\cdots 9. \tag{1}$$

The vectors $|\psi_i\rangle$ occupy vertices in the orthogonality graph G and the edges connect the vertices occupied by mutually orthogonal vectors as shown in Fig. 1.

As per non-contextual hidden variable theories, one can consider a pre-assignment of the dichotomous measurement outcomes (0 or 1) to each of the projection operators Π_i. The projection operators obey the orthogonality relations depicted in Graph G, due to which no two connected vertices in Graph G can simultaneously be assigned the value 1. Thus the sum total of the maximum value of the measurement outcomes that can be obtained from Graph G is 3. Therefore, repeated projective measurements of Π_i over an ensemble of identically prepared single qutrit states yield the following inequality:

$$\sum_{i=1}^{9} \langle \Pi_i \rangle \leq 3. \tag{2}$$

Introducing dichotomous observables $A_i = I - 2\Pi_i$ with eigen values ± 1 associated with projection operators Π_i and by using the inequality in (2) we obtain

$$\sum_{i=1}^{9} \langle A_i \rangle \geq 3. \tag{3}$$

Measurement outcomes of A_i and A_j which are connected to each other by an edge in the graph G are mutually exclusive and can be measured simultaneously. The edge can be thought of as defining a context, and each A_i occurs in more than one context.

Pre-assignment of the measurement outcomes to each of these observables on the basis of a non-contextual hidden variable theory does not consider the joint probability distributions of all those operators which are being co-measured. Considering the non-contextual pre-assignments (as before), (2) is reformulated in terms of the contexts, represented as the correlations between the compatible observables A_i, A_j sharing an edge and leads to the following inequality:

$$\sum_{i,j \in E(G)} \langle A_i A_j \rangle + \langle A_9 \rangle \geq -4 \tag{4}$$

The inequalities in (2) and (3) represent single-qutrit contextuality inequalities in terms of the set of nine measurements and (4) represents the contextuality inequality based on contexts defined in the graph G. The violation of these inequalities indicates the contextual nature of a single-qutrit state and can be experimentally tested by performing repeated measurements of operators A_i on an ensemble of identically prepared qutrit states. There is no unique set of nine measurements that test the contextual nature of every single-qutrit state. However, corresponding to every single-qutrit state (except the maximally mixed state), one can always find a set of nine projectors that reveal its contextuality [8].

2.1. Reformulation of contextuality inequalities in terms of traceless operators

In this subsection we turn to constructing tests to verify qutrit contextuality on an NMR quantum information processor. Since, in NMR we can measure only traceless observables, we need to reformulate the inequalities developed in (2), (3) and (4) in terms of traceless observables. A natural set of traceless observables is provided by the eight Gell-Mann or Λ matrices [19] and they have been used for qutrit analysis earlier [20]. In the basis spanned by the eigen states of S_z, namely the states $\{|+1\rangle, |0\rangle, |-1\rangle\}$ (which we will follow in the rest of this paper), these matrices are given by:

$$\Lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Lambda_2 = \begin{pmatrix} 0 & -\iota & 0 \\ \iota & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\Lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \Lambda_5 = \begin{pmatrix} 0 & 0 & -\iota \\ 0 & 0 & 0 \\ \iota & 0 & 0 \end{pmatrix} \Lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$
$$\Lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\iota \\ 0 & \iota & 0 \end{pmatrix} \quad \Lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{5}$$

At this stage we make a choice of a particular set of nine vectors $|\psi_j\rangle$, $j = 1\cdots 9$, occupying the vertices of the graph G in Fig. 1. The vectors that we choose are given in the S_z basis as:

$$|\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |\psi_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, |\psi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$
$$|\psi_4\rangle = \begin{pmatrix} 0 \\ \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{2}} \end{pmatrix}, |\psi_5\rangle = \begin{pmatrix} \sqrt{\frac{1}{3}} \\ 0 \\ -\sqrt{\frac{2}{3}} \end{pmatrix}, |\psi_6\rangle = \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \\ 0 \end{pmatrix},$$
$$|\psi_7\rangle = \begin{pmatrix} \sqrt{\frac{1}{2}} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, |\psi_8\rangle = \begin{pmatrix} \sqrt{\frac{1}{2}} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, |\psi_9\rangle = \begin{pmatrix} \sqrt{\frac{1}{2}} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{6}$$