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Sign change of the surface energy of a two-component superconductor

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ABSTRACT

It is shown that the sign of the surface energy of a two-component superconductor is determined not only by the Ginzburg–Landau parameters of two superconducting components, but also by a temperature-independent parameter κ_ξ, which is defined as the ratio of the coherence lengths of two components.

It is well known that a conventional superconductor can be categorized as type-1 or type-2, depending on its behavior under a magnetic field. The criterion that determines whether a superconductor is of type-1 or type-2 is the Ginzburg–Landau (GL) parameter κ [1]. It is defined as the ratio of the penetration depth λ over the coherence length ξ, $\kappa = \lambda/\xi$, and determines the sign of the surface energy of a superconductor. The critical value $\kappa_c = 1/\sqrt{2}$ represents the demarcation line between type-1 ($\kappa < \kappa_c$) superconductor which has positive surface energy and type-2 ($\kappa > \kappa_c$) superconductor, which has negative surface energy [2].

Motivated by recent interest in multi-component superconductors [3,4], here we study one-dimensional superconducting-normal boundary in an abstract two-component system. We show that, the sign of the surface energy of such system is determined not only by the Ginzburg–Landau parameters κ_i ($i = 1, 2$) of two superconducting components, but also by a temperature-independent parameter κ_ξ, which is defined as the ratio of the coherence lengths of two components: $\kappa_\xi = \xi_1/\xi_2$. Further, we identify the sign changes of the surface energy of type-1 + type-2 and type-2 + type-2 materials due to the variation of these three parameters.

We start with a general system in which two superconducting components coexist. The GL free energy density of the system is

$$f_s = f_{n0} + \sum_{i=1}^{2} \frac{\hbar^2}{2m_i^*}\left|\left(\nabla - \frac{ie_i^*}{\hbar c}\mathbf{A}\right)\Psi_i\right|^2 + V\left(|\Psi_{1,2}|^2\right) + \eta\left(\Psi_1^*\Psi_2 + \Psi_1\Psi_2^*\right) + \frac{1}{8\pi}(\nabla\times\mathbf{A})^2, \quad (1)$$

where f_{n0} is the free energy density of the body in the normal state in the absence of the magnetic field, $V(|\Psi_i|^2) = a_i|\Psi_i|^2 + b_i|\Psi_i|^4/2$ ($i = 1, 2$). η is a coefficient characterizes Josephson coupling between two superconducting components. In the following we do not consider coupling effect and set $\eta = 0$. We also assume that the effective mass m_i^* and charge e_i^* of two components are equal: $m_i^* = m^*$, $e_i^* = e^*$. There are four characteristic lengths: the penetration depth λ_i and coherence length ξ_i for each component are given by: $\lambda_i = (m^*c^2/4\pi e^{*2}\Psi_{i0}^2)^{1/2}$, $\xi_i = \hbar/(2m^*|a_i|)^{1/2}$, where $\Psi_{i0} = (-a_i/b_i)^{1/2}$. The thermodynamic critical magnetic field of the individual component is $H_{ct(i)} = \Phi_0/(2\sqrt{2}\pi\lambda_i\xi_i)$, where $\Phi_0 = hc/e^*$ is the flux quantum. The magnetic field penetration depth and the thermodynamic critical magnetic field of the system (1) are: $\lambda = (1/\lambda_1^2 + 1/\lambda_2^2)^{-1/2}$, $H_{ct} = (H_{ct(1)}^2 + H_{ct(2)}^2)^{1/2}$. Notice that $\lambda < \min(\lambda_1, \lambda_2)$, $H_{ct} > \max(H_{ct(1)}, H_{ct(2)})$.

Let us consider a plane interface between normal (n) and superconducting (s) phases in a two-component superconductor, taking the interface as the yz-plane and the x-axis into the s phase. The surface energy α_{ns} is defined as, under the thermodynamic critical magnetic field $\mathbf{H}_{ct} = H_{ct}\hat{z}$, the Gibbs energy difference between the n, s transitional state and the fully normal state (or fully super-

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conducting state since these must be equal) of the superconductor with unit cross-section:

$$\alpha_{ns} = \int_{-\infty}^{\infty} dx \left\{ \frac{\hbar^2}{2m^*} \sum_{i=1}^{2} \left| \left(\nabla - \frac{ie^*}{\hbar c} \mathbf{A} \right) \Psi_i \right|^2 + V\left(|\Psi_{1,2}|^2\right) + \frac{1}{8\pi} (\mathbf{H}_{ct} - \nabla \times \mathbf{A})^2 \right\}. \tag{2}$$

The integrand vanishes, both within the n phase ($x \to -\infty$), where $\Psi_i = 0$ and $\nabla \times \mathbf{A} = \mathbf{H}_{ct}$, and within the s phase ($x \to \infty$), where $\Psi_i = \Psi_{i0}$ and $\nabla \times \mathbf{A} = 0$. Now the distribution of all quantities depends only on the coordinate x. This fact enables us to choose gauge potential as $\mathbf{A} = (0, A_y(x), 0)$. Then the order parameters Ψ_i can be taken real. We shall use the dimensionless quantities as following: $\rho \equiv x/\lambda$, $\psi_1 \equiv \Psi_1/\Psi_{10}$, $\psi_2 \equiv \Psi_2/\Psi_{20}$, $A \equiv |\mathbf{A}|/H_{ct}\lambda$, $A' = B \equiv |\nabla \times \mathbf{A}|/H_{ct}$. Then the expression (2) becomes $\alpha_{ns} = (H_{ct}^2\lambda/8\pi) \int_{-\infty}^{\infty} d\rho \{\sum_{i=1}^{2} (H_{ct(i)}/H_{ct})^2 [2(\xi_i/\lambda)^2 \psi_i'^2 + (H_{ct}/H_{ct(i)})^2 (\lambda/\lambda_i)^2 A^2 \psi_i^2 - 2\psi_i^2 + \psi_i^4] + (A'-1)^2\}$. *It can be verified that all coefficients in the integrand can be represented as functions of three dimensionless temperature-independent parameters:* $\kappa_1 \equiv \lambda_1/\xi_1, \kappa_2 \equiv \lambda_2/\xi_2, \kappa_\xi \equiv \xi_1/\xi_2$.[1] And the surface energy (2) can be rewritten as:

$$\alpha_{ns} = \frac{H_{ct}^2\lambda}{8\pi} \int_{-\infty}^{\infty} d\rho \left\{ \sum_{i=1}^{2} \frac{C_i}{B_i} \left[2A_i \psi_i'^2 + \left(B_i A^2 - 2\right)\psi_i^2 + \psi_i^4 \right] + \left(A' - 1\right)^2 \right\}, \tag{3}$$

where $A_1 = 1/\kappa_1^2 + \kappa_\xi^2/\kappa_2^2$, $B_1 = (\kappa_2^2 + \kappa_1^2\kappa_\xi^4)/(\kappa_2^2 + \kappa_1^2\kappa_\xi^2)$, $C_1 = \kappa_2^2/(\kappa_2^2 + \kappa_1^2\kappa_\xi^2)$, $A_2 = 1/\kappa_2^2 + 1/\kappa_1^2\kappa_\xi^2$, $B_2 = (\kappa_2^2 + \kappa_1^2\kappa_\xi^4)/[\kappa_\xi^2(\kappa_2^2 + \kappa_1^2\kappa_\xi^2)]$, $C_2 = \kappa_1^2\kappa_\xi^2/(\kappa_2^2 + \kappa_1^2\kappa_\xi^2)$. The GL equations of motion following from the free energy (1) are:

$$\begin{aligned} A_1\psi_1'' &= \frac{B_1}{2} A^2\psi_1 - \psi_1 + \psi_1^3, \\ A_2\psi_2'' &= \frac{B_2}{2} A^2\psi_2 - \psi_2 + \psi_2^3, \\ A'' &= \left(C_1\psi_1^2 + C_2\psi_2^2\right)A, \end{aligned} \tag{4}$$

with boundary conditions: $\psi_1(-\infty) = \psi_2(-\infty) = 0$, $A'(-\infty) = 1$, $\psi_1(\infty) = \psi_2(\infty) = 1$, $A'(\infty) = 0$. The surface energy α_{ns} is obtained from the substitution of field variables ψ_1, ψ_2, A that satisfy the GL equations (4) into (3).

It is clear from (3) and (4) that the sign of the surface energy is determined by three independent dimensionless parameters: κ_1, κ_2, κ_ξ. If these three parameters are known for a material considered, one can obtain the value of $\alpha_{ns}/(H_{ct}^2\lambda/8\pi)$ from the substitution of ψ_1, ψ_2, A that satisfy (4) into (3), then the sign of the surface energy is identified.

When the coherence lengths of two components are equal: $\xi_1 = \xi_2$, i.e., $\kappa_\xi = 1$, it is easily verified that Eqs. (4) have the first integral:

$$\frac{\kappa_2^2}{\kappa_1^2 + \kappa_2^2} \left[2\left(\frac{1}{\kappa_1^2} + \frac{1}{\kappa_2^2} \right)\psi_1'^2 + \left(2 - A^2\right)\psi_1^2 - \psi_1^4 \right] + \frac{\kappa_1^2}{\kappa_1^2 + \kappa_2^2} \left[2\left(\frac{1}{\kappa_1^2} + \frac{1}{\kappa_2^2} \right)\psi_2'^2 + \left(2 - A^2\right)\psi_2^2 - \psi_2^4 \right] + A'^2 = 1. \tag{5}$$

[1] In fact here we need at least three parameters to express ratios of four characteristic lengths: λ_1, λ_2, ξ_1, ξ_2. We select κ_1, κ_2, κ_ξ because they have definite physical meaning and help to give an intuitive estimate of the sign of the surface energy.

With (5), the expression (3) becomes $\alpha_{ns} = \frac{H_{ct}^2\lambda}{4\pi} \int_{-\infty}^{+\infty} d\rho \{\frac{2}{\kappa_1^2}\psi_1'^2 + \frac{2}{\kappa_2^2}\psi_2'^2 + A'(A'-1)\}$. The sign of the surface energy can be determined by the procedures described above. Let us remark on several particular cases: In the case $\kappa_1 \gg 1$, $\kappa_2 \gg 1$, the first two terms in the integrand can be neglected and the sign of the surface energy is always negative since $A' \in [0, 1]$. The sign of the surface energy is positive in the opposite case $\kappa_1 \ll 1$, $\kappa_2 \ll 1$. If one component is of extreme type-2, while the other component is of extreme type-1, i.e., $\kappa_1 \gg 1$, $\kappa_2 \ll 1$. The contribution of the first term in the integrand can be neglected and the sign of the surface energy is positive.

Let us now study general cases in which there is disparity in coherence lengths between two components: $\xi_1 \neq \xi_2$, i.e., $\kappa_\xi \neq 1$. We then need to solve Eqs. (4) with given boundary conditions numerically. Before a detailed numerical work is undertaken, we first analyze the problem qualitatively. We note that in the integrand in (2) only the second term $V(|\Psi_{1,2}|^2)$ contributes the negative value to the surface energy. The distance scale over which the condensates tends to its expectation value Ψ_{i0} is of order $\sim \xi_i$. $V(|\Psi_i|^2)$ decreases from 0 to $a_i|\Psi_{i0}|^2 + b_i|\Psi_{i0}|^4/2 = -H_{ct(i)}^2/(8\pi)$ in the same range. The length scale over which the magnetic field decays is $\sim \lambda$. The last term of integrand in (2) $\frac{1}{8\pi}(\mathbf{H}_{ct} - \nabla \times \mathbf{A})^2$ increases from 0 to $\frac{1}{8\pi}H_{ct}^2$ in this range. The similar term in the surface energy expression when only single superconducting component exists increases from 0 to $\frac{1}{8\pi}H_{ct(i)}^2$ in a range $\sim \lambda_i$. Since $\lambda < \min(\lambda_1, \lambda_2)$, $H_{ct} = (H_{ct(1)}^2 + H_{ct(2)}^2)^{1/2}$, the integral value of the last term in integrand in (2) is positive and is much larger than the sum of the integral values of similar terms in the surface energy expressions when single component exists. We then conclude that there is a trend of increase in the surface energy of a two-component superconductor comparing to the sum of the surface energy of the single component cases. The detailed numerical simulations below confirm this idea.

To explore the concrete behavior of the sign of the surface energy with three parameters κ_1, κ_2, κ_ξ, we then search for the numerical solutions of the GL equations. (4). And we really identified the sign change of the surface energy due to the variation of these three parameters. There are three cases:

Case 1. $\kappa_1 < 1/\sqrt{2}$, $\kappa_2 < 1/\sqrt{2}$, i.e., two components are both of type-1. As we have shown, there is a trend of increase in the surface energy of a two-component superconductor comparing to the sum of the surface energy of the single component cases. Since the sign of the surface energy of a type-1 material is always positive, we then conclude that the system has positive surface energy.

Case 2. $\kappa_1 > 1/\sqrt{2}$, $\kappa_2 < 1/\sqrt{2}$, i.e., the first component is of type-2, while the second is of type-1. When the third parameter $\kappa_\xi \gg 1$, i.e., $\xi_1 \gg \xi_2$, thus $\lambda_1 > \xi_1/\sqrt{2} \gg \xi_2 > \lambda_2$, $\lambda = (1/\lambda_1^2 + 1/\lambda_2^2)^{-1/2} \approx \lambda_2 < \xi_2$. Then the integral value of the last term in integrand in (2) $\frac{1}{8\pi}(\mathbf{H}_{ct} - \nabla \times \mathbf{A})^2$ gains ascendancy over that of the second term $V(|\Psi_{1,2}|^2)$, and the sign of the surface energy tends to be positive. On the other hand, when $\kappa_\xi \ll 1$, i.e., $\xi_1 \ll \xi_2$, penetration depth λ may fall into the region $\xi_1 < \lambda < \xi_2$, and the sign of the surface energy can take negative. We then conclude that there is a critical value $\kappa_{\xi c}$ at which the surface energy vanishes. As an example, we show in Fig. 1 the sign change of the surface energy of a two-component superconductor with $\kappa_1 = 6.0$, $\kappa_2 = 0.5$. It is clear that the sign of the surface energy is negative when $\kappa_\xi < \kappa_{\xi c} = 0.35$. Generally, for fixed $\kappa_1 > 1/\sqrt{2}$, $\kappa_2 < 1/\sqrt{2}$, the critical value $\kappa_{\xi c}$ can be determined using numerical method as shown above. However, a rough estimate of the upper limit of the critical value $\kappa_{\xi c}$