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Discussion

Dependence of plasma wake wave amplitude on the shape of Gaussian chirped laser pulse propagating in a plasma channel

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ABSTRACT

The generation of longitudinal electrostatic plasma waves (wakefields) due to the propagation of a chirped laser pulse through a parabolic plasma channel is studied. The wakes generated by a temporally symmetric Gaussian laser pulse are compared with those generated by asymmetric one. The main interest in this paper is to investigate the effects of a laser pulse shape with of sharp rising and slow falling time scales on the excited wakefield amplitude. Moreover, positive, negative and un-chirped laser pulses are employed in numerical codes to evaluate the influence of the initial chirp on wakefield excitation. Numerical results showed that for an appropriate laser pulse length compared with the plasma wavelength, the wakefield amplitude can be enhanced for a positively chirped asymmetric Gaussian laser pulse with a fast rise time.

1. Introduction

The interaction of high power lasers with plasma has recently received worldwide research interest because of its wide-ranging applications. One of the most important application is in laser wakefield electron acceleration [1–3]. The laser wakefield acceleration (LWFA) scheme is usually considered a mechanism for electron beam acceleration using available table-top lasers. Medical radiotherapy, cancer treatment and nuclear reactions are some applications of electron accelerators [4]. A high amplitude electron plasma wave can be generated by the effect of the ponderomotive force associated with the envelope of laser pulse propagating through an under-dense plasma. The wake wave can travel at the group velocity of the laser pulse, close to the speed of light, in order to accelerate particles up to relativistic velocities. In 2004, Khachatryan *et al.* showed the possibility to increase the wakefield amplitude which is excited by a chirped laser pulse [5]. In a chirped laser pulse, the laser frequency changes over the duration of the pulse. There are numerous investigations dealing with the effect of chirp on laser pulse interaction with charged particles and plasma [5–12]. In our previous work, we showed that a positively chirped laser pulse could enhance the amplitude of generated wakefield in a plasma channel [12]. Moreover, it was obtained that the temporal shape of the laser pulse profile can significantly affect the plasma wave growth [13,14]. Zhang et al. investigated the pulse profile effect of a plane wave laser on the wakefield generation in a homogeneous plasma [8]. The main purpose of this paper is to investigate the combined effect of the temporal shape and initial chirp of laser pulse on the amplitude of the excited wakefield due to propagation of chirped Gaussian laser pulses through a parabolic plasma channel. The plasma channel has been used to achieve optical guiding and extended propagation of the laser pulse over many Rayleigh lengths, which can affect the production of a stronger wake wave. In the previous work, we only investigated the initial chirp effect of a symmetric Gaussian laser pulse on the generated wakefield amplitude with taking into account group velocity dispersion (GVD) and nonlinear relativistic effects [12]. In order to learn how laser pulse shape can affect wake wave amplitude, we have considered a symmetric Gaussian laser pulse as well as an asymmetric one with a fast/slow rise time, in this paper. Moreover, in order to compare the amplitude of the excited wake wave, three different initial chirps for each of them have been considered. Usually, such asymmetric laser pulses can be generated in a fairly simple experimental setup in chirped-pulse amplification (CPA) laser systems by detuning the laser pulse compressor gratings [15,16]. In this work, a set of coupled equations governing the evolution of the laser parameters and plasma wave potential is derived using source dependent expansion (SDE) method [17]. The plasma density inhomogeneity (associated with the plasma channel), the group velocity dispersion and nonlinearities arising from plasma waves and relativistic effect are employed

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to derive the equations. The combined effect of Gaussian pulse shape and initial chirp type on the electric wakefield amplitude in a parabolic plasma channel, considering the relativistic and GVD effects, has been investigated for the first time in this paper. The results show that the amplitude of wake wave is greatly enhanced in the positive chirped Gaussian laser pulse with a fast rise time case.

This paper is organized as follows: In Sec. 2, the analytical investigation of laser pulse propagation through the plasma channel is considered. The discussion and numerical results are presented in Sec. 3, and the summary and conclusions will be given in Sec. 4.

2. Analysis of laser pulse propagation

In this section, general nonlinear equations describing the propagation of a laser pulse in a preformed plasma channel are derived. The electric field of laser pulse propagating in a fully ionized plasma is governed by the wave equation [18]

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}(\mathbf{r},t) = \frac{4\pi}{c^2}\frac{\partial \mathbf{J}_p(\mathbf{r},t)}{\partial t}, \tag{1}$$

where, $\mathbf{E}(\mathbf{r},t)$ is the electric field of the laser pulse, $\nabla^2 = \nabla_\perp^2 + \partial^2/\partial z^2$ is Laplace operator, z is axial propagation direction and $\mathbf{J}_p(\mathbf{r},t)$ is plasma current density. The plasma current density consists of a linear and nonlinear contributions, i.e., $\mathbf{J}_p = \mathbf{J}_{pL} + \mathbf{J}_{pNL}$, where $\mathbf{J}_{pL}$ is the first order in the electric field, $\mathbf{E}$ and $\mathbf{J}_{pNL}$ is nonlinear in $\mathbf{E}$. The linear and nonlinear parts of the plasma current density are given by, $\partial \mathbf{J}_{pL}(\mathbf{r},t)/\partial t = \omega_p^2(r)\mathbf{E}(\mathbf{r},t)/4\pi$ and $\partial \mathbf{J}_{pNL}(\mathbf{r},t)/\partial t = \omega_p^2(r)\,(\delta n/n_0 - \delta m/m)\,\mathbf{E}(\mathbf{r},t)/4\pi$, respectively [12, 19], where, $\omega_p(r) = [4\pi n_p(r)e^2/m]^{1/2}$ is the plasma frequency, $n_p(r)$ is the plasma density, δn is the plasma density perturbation associated with the wakefield and $\delta m/m = |b|^2/4$ is change of the electron's mass due to relativistic effects that $|b|$ is the normalized amplitude of the electric field. Electric field of a laser beam is given by $\mathbf{E}(\mathbf{r},t) = (\hat{\mathbf{e}}_x/2)E(\mathbf{r},t)e^{i(k_0 z - \omega_0 t)} + c.c.$, where, k_0 is wave number in vacuum, ω_0 is the frequency in the center of the pulse, $\hat{\mathbf{e}}_x$ is a unit vector in the x-direction denoting the polarization and $c.c.$ denotes the complex conjugate. Taking a Fourier transform of Eq. (1), by considering Gaussian beams, gives [19]

$$\left(\nabla^2 + \frac{4}{r_0^2} + \frac{\omega^2}{c^2}n_L^2(r,\omega)\right)\hat{\mathbf{E}}(\mathbf{r},\omega) = \frac{\omega_p^2(r)}{c^2}\left[\frac{\delta n}{n_0} - \frac{|b|^2}{4}\right]\hat{\mathbf{E}}(\mathbf{r},\omega), \tag{2}$$

where $n_L(r,\omega) = [1 - (\omega_p(r)/\omega)^2 - (2c/r_0\omega)^2]^{1/2}$, is the linear part of the total refractive index and r_0 is the beam waist of the laser. We assume that the plasma is cold, initially nonuniform, and noncollisional. The ions are also considered to be fixed, immobile and there is no background magnetic field applied to the plasma. An extended propagation of a laser pulse over many Rayleigh lengths, necessary to create a long acceleration distance and high energy electrons, can be achieved by the use of guiding structures such as plasma channels. The pulse remains in the channel over the entire length without diffraction and creates a strong wakefield which can accelerate electrons. The plasma density profile of a parabolic plasma channel is of the form $n(r) = n_0 + \Delta n(r/R_{ch})^2$, where $n_0 = n(r=0)$ is the plasma density on the axis of the channel which is constant along the axis, R_{ch} and Δn are the channel radius and channel depth, respectively. Using Eq. (2) and some mathematics operations, the equation governing the complex amplitude of the laser envelope can be written as [12]

$$\left\{\nabla_\perp^2 + 2ik_0\frac{\partial}{\partial\eta} - k_0\beta_2 v_g^2\frac{\partial^2}{\partial\xi^2} + \frac{4}{r_0^2} - 4\pi n_0 r_e\left[\frac{n_p(r)}{n_0}\left(1 + \frac{\delta n}{n_0} - \frac{|b|^2}{4}\right) - 1\right]\right\}E(r,\xi,\eta) = 0. \tag{3}$$

Where, $\nabla_\perp^2 = (1/r)\partial/\partial r + \partial^2/\partial r^2$ is the transverse part of the Laplace operator, $r_e = e^2/mc^2$ is the classical electron radius, $\eta = z$, $\xi = z - v_{ph}t$ is the retarded coordinate, v_{ph} is the phase velocity of the wake wave which equals the group velocity of the laser pulse propagating in the under-dense plasma, and $\beta_2 = -(\omega_{p0}^2 + 4c^2/r_0^2)/\omega_0^3 c$ is related to group velocity dispersion (GVD), which here is constant along the propagation direction. Notice that for a homogeneous plasma ($\Delta n = 0$), it is enough to take $n_p(r) = n_0$, in Eq. (3). This equation has been solved for a Gaussian laser beam using source-dependent expansion (SDE) method [17]. In this investigation, a pulse which propagates through the plasma in the z-direction and polarized linearly in the x-direction is considered. The normalized amplitude of the chirped Gaussian pulse is given by

$$b(r,\xi,\eta) = B(\eta)e^{i\psi(\eta)}e^{-[1+i\alpha(\eta)]r^2/r_s^2(\eta)}e^{-[1+i\beta(\eta)]\xi^2/L_p^2(\eta)}, \tag{4}$$

where, $B(\eta)$ is the unit-less magnitude of the electric field, $\psi(\eta)$, $r_s(\eta)$ and $L_p(\eta)$ are the phase, spot size and length of laser pulse, respectively. $\alpha(\eta)$ is a dimensionless parameter that is inversely proportional to the wavefront radius of curvature and $\beta(\eta)$ is a dimensionless chirp parameter of the laser beam. Here, $\beta > 0$ means a positive chirp, $\beta < 0$ means a negative chirp and $\beta = 0$ means that the laser pulse is un-chirped. In Eq. (4), the length of the laser pulse is described as $L_p = \sigma_r L_0$ for $0 < \xi < L_0/2$ and $L_p = \sigma_f L_0$ for $L_0/2 < \xi < L_0$, where L_0 is the full length of the laser pulse, σ_r $(0 < \sigma_r < 1)$ and $\sigma_f (= 1 - \sigma_r)$ represent the Gaussian pulse rise and fall edge unit-less coefficients, respectively. In the present study, for a laser pulse length, three different Gaussian pulse shapes are considered: a symmetric pulse ($\sigma_r = \sigma_f$) and two asymmetric pulses with a sharp rising front ($\sigma_r < \sigma_f$) and a gentle rising front ($\sigma_r > \sigma_f$). In the SDE method the equation of wave propagation is written in the form

$$\left[\nabla_\perp^2 + 2ik_0\frac{\partial}{\partial\eta}\right]\mathbf{b}(r,\xi,\eta) = M(r,\xi,\eta)\mathbf{b}(r,\xi,\eta), \tag{5}$$

where, using Eq. (3), the $M(r,\xi,\eta)$ is given by

$$M(r,\xi,\eta) = 4k_0 v_g^2\beta_2\left[-\frac{1+i\beta(\eta)}{2L_p^2(\eta)} + \left(\frac{1+i\beta(\eta)}{L_p^2(\eta)}\xi\right)^2\right] - \frac{4}{r_0^2} + 4\pi n_0 r_e\left[\frac{n(r)}{n_0}\left(1 + \frac{\delta n}{n_0} - \frac{|b|^2}{4}\right) - 1\right]. \tag{6}$$

Following the standard SDE procedure [17] the equations governing the laser pulse parameters such as spot size r_s, laser pulse length L_p, B, ψ, α and β can be given by

$$\frac{\partial^2 r_s(\eta)}{\partial\eta^2} = \frac{\lambda_0^2}{\lambda_{p0}^2}\left[\frac{4}{r_s^3(\eta)} - \frac{\Delta n}{n_0}\left(1 + \frac{\delta n}{n_0}\right)\frac{r_s(\eta)}{R_{ch}^2} - \frac{B^2(\eta)}{8r_s(\eta)}\right], \tag{7}$$

$$\frac{\partial^2 L_p(\eta)}{\partial\eta^2} = \frac{\lambda_0}{\lambda_{p0}}\left[\frac{\beta_2}{4}\frac{B^2(\eta)}{L_p(\eta)}\left(1 + \frac{\Delta n}{n_0}\frac{r_s^2(\eta)}{4R_{ch}^2}\right)\right] + \frac{4\beta_2^2}{L_p^3(\eta)}, \tag{8}$$

$$\frac{\partial B(\eta)}{\partial\eta} = -\frac{B(\eta)}{r_s(\eta)}\frac{\partial r_s(\eta)}{\partial\eta} - \frac{B(\eta)}{2L_p(\eta)}\frac{\partial L_p(\eta)}{\partial\eta}, \tag{9}$$

$$\frac{\partial\psi(\eta)}{\partial\eta} = \frac{2\lambda_0}{\lambda_{p0}}\left[\frac{1}{r_0^2} - \frac{1}{r_s^2(\eta)} + \left(\frac{B^2(\eta)}{64}\left(3 + \frac{\Delta n}{n_0}\frac{r_s^2(\eta)}{2R_{ch}^2}\right) - \frac{\delta n}{4n_0}\right)\right] + \frac{\beta_2}{L_p^2(\eta)}, \tag{10}$$