Physics Letters A 380 (2016) 1571–1576

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Thermal entanglement and teleportation in a dipolar interacting system

C.S. Castro [a,b,*], O.S. Duarte [c], D.P. Pires [c], D.O. Soares-Pinto [c], M.S. Reis [a]

[a] Instituto de Física, Universidade Federal Fluminense, Av. Gal. Milton Tavares de Souza s/n, Gragoatá, 24210-346 Niterói, RJ, Brazil
[b] Centro de Formação de Professores, Universidade Federal do Recôncavo da Bahia, Av. Nestor de Mello Pita, n. 535, 45.300-000 Amargosa, BA, Brazil
[c] Instituto de Física de São Carlos, Universidade de São Paulo, P.O. Box 369, São Carlos, 13560-970 SP, Brazil

A R T I C L E I N F O

Article history:
Received 11 July 2015
Received in revised form 2 March 2016
Accepted 3 March 2016
Available online 8 March 2016
Communicated by L. Ghivelder

Keywords:
Quantum entanglement
Quantum teleport
Dipolar interaction

A B S T R A C T

Quantum teleportation, which depends on entangled states, is a fascinating subject and an important branch of quantum information processing. The present work reports the use of a dipolar spin thermal system as a noisy quantum channel to perform quantum teleportation. Non-locality, tested by violation of Bell's inequality and thermal entanglement, measured by negativity, shows that for the present model all entangled states, even those that do not violate Bell's inequality, are useful for teleportation.

1. Introduction

Quantum mechanics is characterized by its counter-intuitive concepts as, for example, quantum entanglement, whose importance in modern physics has stimulated intensive research of several quantum systems [1]. Such quantum correlation implies that the subsystems cannot be thought of as individual objects but as a single inseparable quantum system, i.e., their quantum state cannot be factorized into single states. In its strongest form, when the components of the system are far apart, this correlation implies that the properties of the system are distributed among the parts and this characterizes the observation of non-local phenomena certified by violation of Bell inequalities [2], which cannot be explained by a classical theory.

In terms of quantum information purposes, quantum entanglement is an important resource and, consequently, its quantification is fundamental. While entanglement for pure states is completely understood and equivalent to non-locality [2], for mixed states many forms to quantify entanglement were proposed and it was shown that it is not always equivalent to non-local properties [3]. Still in the case of mixed states, for some bipartite systems several measures, such as negativity, are available to analytically quantify entanglement [4].

Concerning usefulness of entangled states, it was showed by Bennett et al. [5] that a bipartite maximally entangled particle state can be used as communication channel to promote the quantum teleportation of a unknown state of a third qubit. Such protocol can be seen as a corner stone of quantum information processing because it was one of the first to show the usefulness of quantum entanglement for quantum communication. In that work the authors noted that states that are not maximally entangled can still be used for teleportation but with reduced efficiency/fidelity. Nowadays, this communication protocol is known as standard teleportation protocol.

Regarding this procotol, one question was asked: Is non-locality needed for teleportation? The answer is no. There are some entangled mixed states that do not violate a Bell inequality that still can be used as quantum communication channel for teleportation of an unknown state [6,7]. Also, it was showed that such protocol with mixed states as resource is equivalent to a generalized depolarizing channel [8]. This is an important result because stabilishes that the quantification of the success of teleportation for this protocol can be seen as a quantification of entanglement.

Although very useful, entanglement is a very fragile quantum property. The disappearance of this quantum correlation can be caused by different kinds of sources acting on the system like temporal evolution [9–13] or variation of temperature [14–21] in

* Corresponding author.
E-mail address: ccastro@if.uff.br (C.S. Castro).

http://dx.doi.org/10.1016/j.physleta.2016.03.002
0375-9601/

equilibrium states. So, the understanding of such mechanisms and how they act over the system destroying entanglement is very important in order to keep this quantum advantage of protocols. It is important to note that the case of thermal entanglement, i.e., the entanglement of quantum systems at finite temperature, is one of the links between quantum information and condensed matter areas [22], and consequently, it has been extensively studied by both, theoretical and experimental physicists [23–31] since possible connections to the production of quantum devices based in current technology.

In this work we report the effects of a dipolar interaction between two spins on their degree of entanglement and nonlocality. Also, considering such model a quantum communication channel, we analyze the effects of such interaction over its communication capacity through the teleportation fidelity. Such interaction arises due to the influence of the magnetic field created by one magnetic moment on the site of another magnetic moment [33]. We begin with the model of dipolar interaction and show that, for the case of two coupled spins 1/2, whatever is the ground state, we have the presence of entanglement. For this model we certify non-locality through CHSH inequality and quantify the amount of entanglement using negativity, verifying that our model presents some degree of non-locality and entanglement at a given coupling parameters Δ and ϵ. In addition, we show how the magnetic anisotropies can influence the fidelity of teleportation, which is based on the degree of entanglement of the quantum states involved in the process. We calculate the averaged teleportation fidelity and verify that this quantity has a similar behavior of negativity and violation of Bell's inequality. As expected, such process successfully occurs without need of non-locality of quantum states [7,34–37].

2. The model

The dipolar interaction arises from the magnetic field created by a magnetic moment of a spin $\vec{\mu} = -\mu_B g \vec{S}$ [33], where μ_B is the Bohr's magneton and g the giromagnetic factor, on the site of another spin and is represented by the Hamiltonian

$$H = -\frac{1}{3}\vec{S}_1^T \cdot \overleftrightarrow{T} \cdot \vec{S}_2, \tag{1}$$

where $\overleftrightarrow{T} = \mathrm{diag}(\Delta - 3\epsilon, \Delta + 3\epsilon, -2\Delta)$ is a diagonal tensor, $\vec{S}_j = \{S_j^x, S_j^y, S_j^z\}$ is the spin operator, and Δ and ϵ are the dipolar coupling constants between the spins. These parameters are related to the spatial and relative orientation of the spins [33]. The quantum version of the dipolar interaction has been 'quantizing' by replacing the magnetic moment by the angular momentum operator (see details in Ref. [33]); but the meaning of the physical parameters remains the same. In few words, Δ is the axial parameter (lies on the diagonal of the Hamiltonian operator), and ϵ is the rhombic one (lies on the off-diagonal elements of the Hamiltonian operator). These rule the interaction strength and relative orientation between the angular momenta.

That Hamiltonian can describe a pair of spin 1/2 particles and can be written in a matrix form

$$H = \frac{1}{6}\begin{pmatrix} \Delta & 0 & 0 & 3\epsilon \\ 0 & -\Delta & -\Delta & 0 \\ 0 & -\Delta & -\Delta & 0 \\ 3\epsilon & 0 & 0 & \Delta \end{pmatrix}, \tag{2}$$

with the eigenvalues and eigenvector given by

$$\mathcal{E}_{\Psi^-} = 0, \quad \mathcal{E}_{\Psi^+} = -\Delta/3, \quad \mathcal{E}_{\Phi^\mp} = (\Delta \mp 3\epsilon)/6, \tag{3}$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle), \quad |\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle). \tag{4}$$

Note that the eigenvectors are the four Bell states, the well known family bipartite entangled pure states. This Hamiltonian can be written through the spectral decomposition in terms of its states, i.e., $H = \sum_\alpha \mathcal{E}_\alpha |\alpha\rangle\langle\alpha|$, where $\alpha \in \{\Phi^+, \Phi^-, \Psi^+, \Psi^-\}$.

Let's consider the system at thermal equilibrium at a given temperature T be described by the canonical ensemble $\rho = \mathcal{Z}^{-1} e^{-H/k_B T}$. Here H is the Hamiltonian of the system, k_B the Boltzmann's constant and $\mathcal{Z} = \mathrm{Tr}(e^{-H/k_B T})$ is the partition function. Since ρ is a thermal density operator, the entanglement on this state is called thermal entanglement [38–40]. From using Eq. (2), we obtain the density operator,

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23} & \rho_{22} & 0 \\ \rho_{14} & 0 & 0 & \rho_{11} \end{pmatrix}, \tag{5}$$

where

$$\rho_{11} = \frac{1}{\mathcal{Z}} e^{-\beta\Delta/6}\cosh\left(\frac{\beta\epsilon}{2}\right),$$
$$\rho_{22} = \frac{1}{\mathcal{Z}} e^{\beta\Delta/6}\cosh\left(\frac{\beta\Delta}{6}\right),$$
$$\rho_{23} = \frac{1}{\mathcal{Z}} e^{\beta\Delta/6}\sinh\left(\frac{\beta\Delta}{6}\right),$$
$$\rho_{14} = -\frac{1}{\mathcal{Z}} e^{-\beta\Delta/6}\sinh\left(\frac{\beta\epsilon}{2}\right) \tag{6}$$

and

$$\mathcal{Z} = 2e^{\beta\Delta/6}\cosh\left(\frac{\beta\Delta}{6}\right) + 2e^{-\beta\Delta/6}\cosh\left(\frac{\beta\epsilon}{2}\right). \tag{7}$$

The parameters Δ and ϵ can be experimentally determined since they are within the partition function $\mathcal{Z}$ and, consequently, can be related to the thermodynamical quantities like magnetic susceptibility and heat capacity.

The density operator can also be written in terms of Bell states as $\rho = \sum_\alpha p_\alpha |\alpha\rangle\langle\alpha|$, where $p_\alpha = \mathcal{Z}^{-1} e^{-\mathcal{E}_\alpha/k_B T}$ is the Boltzmann weight. Note that the density matrix of the system is expressed in terms of the four Bell states, which are not possible to be written as a convex sum of the original spin states. Thus, the system will always present some degree of entanglement whatever is the groundstate. This will not happen only when two or more of these states present the same occupation probability. In that case the ground state will be a mixture that produce a separable state.

Generally speaking, the density operator can be written in the Fano form [41]

$$\rho = \frac{1}{4}\left[\mathbb{I}\otimes\mathbb{I} + \vec{r}\cdot\vec{\sigma}\otimes\mathbb{I} + \mathbb{I}\otimes\vec{s}\cdot\vec{\sigma} + \sum_{i,j} c_{ij}\sigma_i\otimes\sigma_j\right], \tag{8}$$

where $\mathbb{I}$ is the 2×2 identity operator, σ_i are the Pauli matrices, $r_j = \langle\sigma_j\otimes\mathbb{I}\rangle$, $s_j = \langle\mathbb{I}\otimes\sigma_j\rangle$, and $c_{ij} = \langle\sigma_i\otimes\sigma_j\rangle$ are spin–spin correlation functions. Considering Eq. (5), the Fano form reduce to

$$\rho = \frac{1}{4}\left(\mathbb{I}\otimes\mathbb{I} + \sum_i c_i\sigma_i\otimes\sigma_i\right), \tag{9}$$

where $r_j = 0$, $s_j = 0$, $c_{ij} = \delta_{ij} c_i$, and

$$c_1 = \frac{2}{\mathcal{Z}}\left[e^{\beta\Delta/6}\sinh\left(\frac{\beta\Delta}{6}\right) - e^{-\beta\Delta/6}\sinh\left(\frac{\beta\epsilon}{2}\right)\right],$$
$$c_2 = \frac{2}{\mathcal{Z}}\left[e^{\beta\Delta/6}\sinh\left(\frac{\beta\Delta}{6}\right) + e^{-\beta\Delta/6}\sinh\left(\frac{\beta\epsilon}{2}\right)\right],$$
$$c_3 = \frac{2}{\mathcal{Z}}\left[-e^{\beta\Delta/6}\cosh\left(\frac{\beta\Delta}{6}\right) + e^{-\beta\Delta/6}\cosh\left(\frac{\beta\epsilon}{2}\right)\right]. \tag{10}$$