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Temperature-dependent transformation thermotics for unsteady states: Switchable concentrator for transient heat flow

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ABSTRACT

For manipulating heat flow efficiently, recently we established a theory of temperature-dependent transformation thermotics which holds for steady-state cases. Here, we develop the theory to unsteady-state cases by considering the generalized Fourier's law for transient thermal conduction. As a result, we are allowed to propose a new class of intelligent thermal metamaterial — switchable concentrator, which is made of inhomogeneous anisotropic materials. When environmental temperature is below or above a critical value, the concentrator is automatically switched on, namely, it helps to focus heat flux in a specific region. However, the focusing does not affect the distribution pattern of temperature outside the concentrator. We also perform finite-element simulations to confirm the switching effect according to the effective medium theory by assembling homogeneous isotropic materials, which bring more convenience for experimental fabrication than inhomogeneous anisotropic materials. This work may help to figure out new intelligent thermal devices, which provide more flexibility in controlling heat flow, and it may also be useful in other fields that are sensitive to temperature gradient, such as the Seebeck effect.

1. Introduction

Heat is one of the most common forms of energy in nature. To control heat flux and temperature gradient is both challenging and important. In recent years, many attempts have been made toward the possibility of manipulating thermal conduction to achieve desired temperature distribution or heat flux patterns [1–3], among which the approach with thermal metamaterials has won great successes.

In 2008, with the aforementioned attempt, Fan et al. [4] proposed steady-state transformation thermotics (or transformation thermodynamics) enlightened by the transformation optics established in 2006 [5,6], which has been successfully extended to other fields, such as acoustics waves [7–9], elastic waves [10], electric currents [11,12], and matter transport [13]. Later, originated from the steady-state and unsteady-state transformation thermotics [4, 14–18], researchers have designed plenty of thermal metamaterials with novel functions [19–27]. This method promises people to manipulate heat flow with more freedom by tailoring thermal conductivities of materials. It is long known that thermal conductivities vary with temperature, meaning a strong nonlinearity in the conduction equation (generalized Fourier equation). The existing theory of transformation thermotics does not take this effect into account until we recently put forward the theory of temperature-dependent transformation thermotics [28]. The new theory not only extended the existing transformation thermotics to a broader range of materials with temperature-dependent conductivities, but also offers a different tool to design devices with novel functions. However, our last work [28] only describes the steady-state cases, namely, the time-independent heat conduction process. Since the change of temperature distribution with time is always important in real applications, it becomes necessary to propose a temperature-dependent transformation thermotics which deals with the unsteady-state conduction equation. This is just an agenda of this work. Then, we take thermal concentrators as an example to show how the unsteady-state temperature-dependent transformation thermotics works. A thermal concentrator can help to raise the temperature gradient in a specific region, which, however, does not affect the pattern of temperature distribution outside the concentrator [19]. Here, we shall propose an intelligent concentrator, which can automatically be switched on or off as environmental temperature changes. Such a concentrator provides a different approach to non-invasively control the temperature gra-

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dient in real time, and thereby may help to adjust the efficiency of thermoelectric effects [29–31] or thermal energy harvesting cell [32] without unexpected interferences to the system. Also, it has other potential applications in heat preservation or energy-saving machines, and can be employed as building blocks in more complicated facilities.

2. Theory and design method

Our recently established theory of temperature-dependent (or nonlinear) transformation thermotics [28] only takes the steady-state thermal conduction into consideration. As a necessary extension, here we apply the theory on transient thermal conduction.

Consider the generalized Fourier's law for transient thermal conduction in n-dimension (without heat source),

$$\frac{\partial \rho c(T) T}{\partial t} = \nabla \cdot [\boldsymbol{\kappa}(T) \cdot \nabla T], \tag{1}$$

where ρ is the density, c is the specific heat capacity, and $\boldsymbol{\kappa}(T)$ is the thermal conductivity tensor which depends on temperature T. We take the multiply $\rho c(T)$ as a whole and assume it to be also temperature-dependent. By expressing Eq. (1) in a curvilinear coordinate system ($x^i, i = 1, \ldots, n$) corresponding to a transformation, we have

$$\frac{\partial \rho c(T) T}{\partial t} = \frac{\partial}{\partial x^i} \kappa^{ij}(T) \frac{\partial}{\partial x^j} T + \Gamma^i_{ik} \kappa^{kj}(T) \frac{\partial}{\partial x^j} T, \tag{2}$$

where Γ^i_{ik} is the Christoffel symbol satisfying

$$\Gamma^i_{ik} = \frac{1}{2} g^{il} \frac{\partial}{\partial x^k} g_{il} = \det(J) \frac{\partial}{\partial x^k} \frac{1}{\det(J)}, \tag{3}$$

where g is the metric tensor, and J is the Jacobian matrix corresponding to the transformation. In order to rewrite Eq. (2) in the physical Cartesian coordinate system (x'^i, $i = 1, \ldots, n$), we perform the variable change from x^i to the Cartesian coordinate x'^i and obtain

$$\begin{aligned} \frac{1}{\det(J)} \frac{\partial \rho c(T) T}{\partial t} &= \frac{\partial \det^{-1}(J) \rho c(T) T}{\partial t} \\ &= \frac{\partial}{\partial x'^i} \left[\frac{J \kappa(T) J^{\mathrm{t}}}{\det(J)} \right]^{ij} \frac{\partial}{\partial x'^j} T, \end{aligned} \tag{4}$$

where J^{t} and $\det(J)$ are respectively the transverse and the determinant of the Jacobian matrix J. We can see that the desired thermal conductivity $\tilde{\kappa}(T)$ is the same as that for the steady-state case

$$\tilde{\kappa}(T) = \frac{J \kappa(T) J^{\mathrm{t}}}{\det(J)}. \tag{5}$$

The additional requirement on the metamaterial for the transient case is that the multiply of its density $\tilde{\rho}$ and heat capacity $\tilde{c}$ should satisfy

$$\tilde{\rho}\tilde{c}(T) = \frac{\rho c(T)}{\det(J)}. \tag{6}$$

This requirement makes at least one of $\tilde{\rho}$ and $\tilde{c}$ a function of the spacial coordinates, and increases the difficulty in designing and realizing the device. Later, we shall present a convenient approach to handle this problem.

It is shown in our previous work [28] that $\tilde{\kappa}(T)$ can be expressed in the following form

$$\tilde{\kappa}(T) = \frac{\tilde{J}(T) \kappa_0 \tilde{J}(T)^{\mathrm{t}}}{\det[\tilde{J}(T)]}, \tag{7}$$

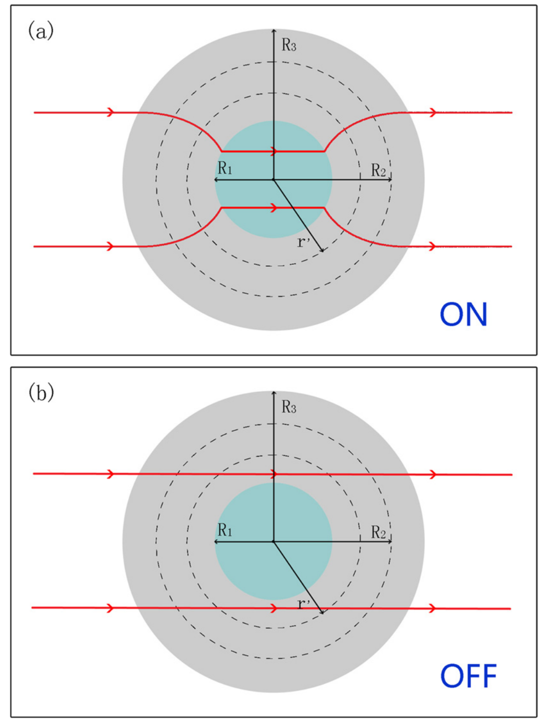

Fig. 1. Schematic graphs of a switchable thermal concentrator when the concentrating effect is switched (a) on or (b) off. The red lines with arrows represent the flow of heat. R_1 and R_3 denote the interior radius and exterior radius, respectively. R_2 and r' are also indicated, whose meaning can be found in the text. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where $\tilde{J}(T)$ is the Jacobian matrix corresponding to a transformation which varies with temperature, and κ_0 is a constant temperature-independent thermal conductivity. Similarly, we can also have

$$\tilde{\rho}\tilde{c}(T) = \frac{\rho_0 c_0}{\det[\tilde{J}(T)]}, \tag{8}$$

where ρ_0 and c_0 are temperature-independent density and heat capacity, respectively. Based on the above expressions, we are able to make a thermal metamaterial behave differently at different environmental temperatures. As a demonstration, we introduce the concept of switchable thermal concentrator.

The function of a thermal concentrator is schematically plotted in Fig. 1(a), where a polar coordinate system (r', θ') is constructed. The device is the gray ring with interior radius R_1 and exterior radius R_3. It guides heat flux (indicated with red lines with arrows) to travel through its inner region ($r' < R_1$), and thus increases the temperature gradient inside it without disturbing the temperature distribution of the background ($r' > R_3$). As researchers have proved [15,19], this function can be achieved by using a transformation, $r' = f(r)$ and $\theta' = \theta$, where

$$r' = f(r) = \begin{cases} r\frac{R_1}{R_2}, & r' < R_1 \\ r\frac{R_3 - R_1}{R_3 - R_2} + R_3 \frac{R_1 - R_2}{R_3 - R_2}, & R_1 \le r' \le R_3 \end{cases}. \tag{9}$$

Here, R_2 is a constant parameter that satisfies $R_1 < R_2 < R_3$. Based on the original idea of a thermal concentrator, the switchable thermal concentrator is a device which only functions at certain temperature range. For example, the device can be designed to function as a normal thermal concentrator when environmental temperature is lower (or higher) than a critical temperature T_c, but have no influence on the field profile when temperature is beyond (or below) T_c.