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Effect of excavated trapped electron distributions on ion-acoustic solitary structures in an electron–positron–ion plasma

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ABSTRACT

Fully nonlinear propagation of ion-acoustic solitary waves in an unmagnetized electron–positron–ion plasma is investigated. A more realistic situation is considered in which electrons interact with the wave potential during its evolution and, follow the vortex-like excavated trapped distribution. The basic properties of large amplitude solitary waves are studied by deriving an energy integral equation involving Sagdeev potential. It is shown that effects of such electron behavior and positron concentration change the maximum values of the Mach number and amplitude for which solitary waves can exist. The small amplitude limit is also investigated by expanding the Sagdeev potential to include third-order nonlinearity of electric potential. In this case, exact analytical solution is obtained which is related to the contribution of the resonant electron to the electron density. It is shown from both highly and weakly nonlinear analysis that the plasma system under consideration supports only compressive solitary waves.

During the last decade the study of nonlinear propagation of electrostatic excitations in electron–positron–ion (e–i–p) plasmas attracted significant attention among researchers [1–9]. Such a plasma can be observed in the inner region of accretion discs in the vicinity of black holes [10], in magnetosphere of neutron stars [11,12], in active galactic cores [13] and even in solar flare plasma [14]. Three components e–i–p plasmas can also be created in the laboratory plasma. It is well known that propagation of a short relativistically strong laser pulse in matter can be accompanied by the formation of e–i–p plasmas due to photo production of pairs during photon scattering by nuclei, etc., [15,16]. Another example is related to plasma in tokamaks and other magnetic confinement systems [17]. In laboratory plasma, over a wide range of parameters the positron life time or annihilation of electrons and positrons, is relatively unimportant [18]. At low temperatures of the order of 1 eV and electron density of 10^{12} cm^{-3}, the observed positron annihilation time is greater than 1 second, which is much larger than the characteristic time scale for the ion-acoustic wave [9].

In contrast to the usual plasma with electrons and positive ions, it has been known that the nonlinear waves in plasmas having positrons behave differently. Linear and nonlinear wave excitations in e–p and e–p–i plasmas have been studied by using different models. For example, Popel et al. [1] investigated nonlinear dynamics of ion-acoustic solitary waves in three component plasmas, whose constituents are free electrons and positrons, and singly charged ions. They showed that the presence of the positron component in a plasma can result in the reduction of the ion-acoustic amplitude. Mahmood et al. [5] studied the large amplitude solitons propagating obliquely with respect to an external magnetic field in a homogeneous magnetized electron–positron–ion plasma. It is found that the amplitude of soliton increases with the presence of positron component, which is an opposite behavior to the previous study of these waves in an unmagnetized plasma.

On the other hand, of particular interest is the case that electrons interact with the ion-acoustic wave potential during its evolution, and therefore can be trapped in the wave potential. In this case, the isothermally distributed electrons often change toward vortex-like electron distributions. Owing to the presence of trapped electrons, e–i–p plasmas are characterized by new, modified properties and conditions for the existence of localized ion-acoustic excitations, as shown in Ref. [7]. Hence the main motivation of the present work is to study the effects of positron concentration and excavated trapped electron distributions (corresponding to an underpopulation of trapped electrons) on the propagation of arbitrary amplitude ion-acoustic solitary waves. Using the pseudopotential approach, we have derived an energy integral equation involving

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a Sagdeev potential. The latter numerically analyzed to study the profiles of the Sagdeev potential and the localized wave properties. In order to confirm the possibility of the ion-acoustic soliton, we also investigated the Sagdeev potential in the small amplitude limit. By considering two asymptotic cases which is related to the contribution of the resonant electrons, we investigate the dependence of the width and the amplitude of solitary waves on the positron concentration. We also show from both highly and weakly nonlinear analysis that the present e–i–p plasma can support only compressive solitary waves.

We consider a one-dimensional, collisionless and unmagnetized three component plasma composed of cold ions, Boltzmann distributed positrons and electrons with excavated trapped particles. The nonlinear propagation of the electrostatic ion-acoustic waves is governed by [7]

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \varphi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = n_e - n_p - (1-p) n_i, \tag{3}$$

where n_i, n_p, n_e, u_i and φ are respectively, the non-dimensional ion number density, the positron number density, the electron number density, the ion fluid velocity and the electrostatic potential. The reference density, speed, length, time and electrostatic potential are respectively, the equilibrium electron number density n_{e0}, the ion sound speed $C_s = (\kappa T_{ef}/m_i)^{1/2}$, $\lambda_{De} = (\epsilon_0 \kappa T_{ef}/n_0 e^2)^{1/2}$, λ_{De}/C_s and $\kappa T_{ef}/e$. Here κ is Boltzmann's constant, $p = n_{p0}/n_{e0}$ is fractional concentration of positrons with respect to electrons in the equilibrium state and, $\sigma = T_{ef}/T_p$ is the temperature ratio between electrons and positrons.

The model used here is based on the assumption of the existence of a vortex-like excavated distribution for electrons which corresponds to an underpopulation of trapped particles. This expression is obtained by the consideration than electrons with higher density that positrons have a great chance to be in resonance with the nonlinear ion-acoustic wave potential. In such environments, the electron number density n_e can be expressed as [19,20]

$$n_e = I(\varphi) + \frac{2}{\sqrt{\pi|\beta|}} W(\sqrt{-\beta\varphi}), \tag{4}$$

where

$$I(\varphi) = \left[1 - \operatorname{erf}(\sqrt{\varphi})\right] e^{\varphi},$$

$$\operatorname{erf}(\sqrt{\varphi}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\varphi}} e^{-t^2}\, dt,$$

$$W(\sqrt{\beta\varphi}) = e^{-\beta\varphi} \int_0^{\sqrt{\beta\varphi}} e^{t^2}\, dt.$$

Eq. (4) shows that electrons could interact nonlinearly with the low-frequency electrostatic potential during its evolution, and therefore can be trapped in the wave potential. Here $\beta = T_{ef}/T_{et}$ is the trapping parameter describing the temperature of the trapped electrons (ratio of free electron temperature to trapped electron temperature). Negative values of β, which we are interested in here, lead to a vortex-like excavated distribution for electrons which corresponds to an underpopulation of trapped particles.

In order to investigate the basic properties of stationary large amplitude ion-acoustic waves, we introduce a single variable $\xi = x - Mt$, where ξ is normalized to λ_{De} and M is the Mach number of the solitary waves with respect to ion-acoustic wave frame for the system. Then, using the steady state condition and imposing the appropriate boundary conditions for localized perturbations, namely, $n_i \to 1$, $u_i \to 0$, $\varphi \to 0$ and $d\varphi/d\xi \to 0$ at $\xi \to \pm\infty$, we reduce Eqs. (1)–(3) to the form of an energy-like integral

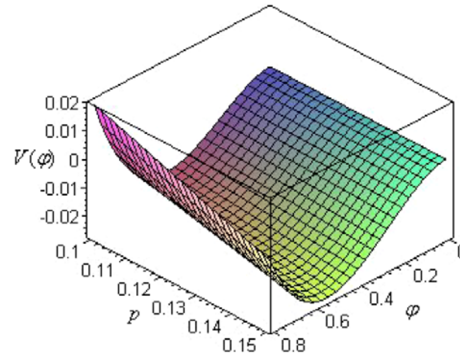

Fig. 1. Behavior of the Sagdeev potential $V(\varphi)$ [represented by Eq. (6)] against the potential φ and positron concentration p for $\beta = -0.5$ and $M = 1.5$.

$$\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2 + V(\varphi) = 0, \tag{5}$$

where $V(\varphi)$ is the Sagdeev pseudopotential [21] and is given by

$$V(\varphi) = 1 - I(\varphi) - \frac{2\sqrt{\varphi}}{\sqrt{\pi}}\left(1 - \frac{1}{\beta}\right) - \frac{2}{\beta\sqrt{\pi|\beta|}} W(\sqrt{-\beta\varphi}) + M^2(1-p)\left[1 - \left(1 - \frac{2\varphi}{M^2}\right)^{\frac{1}{2}}\right] + \frac{p}{\sigma}\left(1 - e^{-\sigma\varphi}\right). \tag{6}$$

Eq. (5) can be analyzed as in classical mechanics, in terms of the properties of the Sagdeev potential $V(\varphi)$. It is obvious that $V(0) = V'(0) = 0$, where primes refer to derivatives with respect to φ. We furthermore require $V''(0) < 0$, causing the origin to be a local unstable maximum and ensuring that a non-zero φ_m (maximum or minimum value of φ) exists for which $V(\varphi_m) = 0$. It is of interest to find out the lower and upper limits of the Mach number M for which solitons exist. The minimum Mach number M_c at which the second derivative of the Sagdeev potential changes its sign can be found as $M_c = \sqrt{(1-p)/(1+p\sigma)}$. The upper limit of M can be obtained by the condition $V(\varphi_c) \geqslant 0$, where $\varphi_c = M^2/2$ is the maximum value of the amplitude for which the ion density is real. Thus, this procedure yields

$$M^2 \geqslant \frac{1}{1-p}\left[I(\varphi_c) - 1 + \frac{2\sqrt{\varphi_c}}{\sqrt{\pi}}\left(1 - \frac{1}{\beta}\right) + \frac{2}{\beta\sqrt{\pi|\beta|}} W(\sqrt{-\beta\varphi_c}) - \frac{p}{\sigma}\left(1 - e^{-\sigma\varphi_c}\right)\right]. \tag{7}$$

The above inequality provides the maximum M, which turns out to depends on the parameters p, σ and β. Adopting the values of $p = 0.2$, $\sigma = 1$ and $\beta = -0.6$ the range of the Mach number can be obtained as $0.81 \leqslant M \precsim 3.02$ for which solitary waves can exist. On the other hand, increasing the positron to electron number density ratio p, for example $p = 0.4$, shows that the range of possible Mach number changes as $0.65 \leqslant M \precsim 2.7$. This indicates that an increase of the positron number density with respect to electron number density leads to the propagation of solitary waves with smaller values of the Mach number.

To examine the possibility of the solitary waves excitation in plasma, we have numerically analyzed the pseudopotential (6) using typical parameters. The free electron temperature and positron