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# Electromagnetic envelope solitons in magnetized plasma

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ABSTRACT

A multiple scales technique is employed to solve the fluid-Maxwell equations describing a weakly nonlinear circularly polarized electromagnetic pulse in magnetized plasma. A nonlinear Schrödinger-type (NLS) equation is shown to govern the amplitude of the vector potential. The conditions for modulational instability and for the existence of various types of localized envelope modes are investigated in terms of relevant parameters. Right-hand circularly polarized (RCP) waves are shown to be modulationally unstable regardless of the value of the ambient magnetic field and propagate as bright-type solitons. The same is true for left-hand circularly polarized (LCP) waves in a weakly to moderately magnetized plasma. In other parameter regions, LCP waves are stable in strongly magnetized plasmas and may propagate as dark-type solitons (electric field holes). The evolution of envelope solitons is analyzed numerically, and it is shown that solitons propagate in magnetized plasma without any essential change in amplitude and shape.



## 1. Introduction

The propagation of intense electromagnetic (EM) pulses through a plasma is associated with a wide variety of interesting relativistic and nonlinear phenomena, such as parametric instabilities [1,2], harmonic generation [3,4], self-focusing [5], generation of intense electric and magnetic fields [6,7], wakefield generation [8], and relativistic EM soliton propagation [9,10]. EM pulse theory is of relevance in a number of applications, including particle acceleration and fast ignition inertial confinement fusion schemes.

Among the nonlinear effects involved in laser plasma interaction, the existence and dynamics of relativistic EM solitons attract great attention both from a fundamental point of view and also due to their possible applications in particle acceleration and fast ignition concept of laser fusion [11,12]. Relativistic EM solitons have been observed in 2D and 3D particle-in-cell (PIC) simulations [13–16] and were recently detected in experiments using a proton imaging technique [6,17]. As shown in [13,15], nearly 25–40 percent of the laser pulse energy goes into the generation of well localized concentrations of electromagnetic energy in the form of soliton or soliton-like structures, which can play an important role in the laser plasma interaction process.

Many different physical effects play a role in the generation of relativistic EM solitons, including dispersion effects due to the finite particle inertia, nonlinearities due to relativistic mass increase as well as ponderomotive effects that redistribute the plasma density. Several analytical and numerical investigations have been carried out, concerning relativistic EM solitons. Theoretical analysis has been performed mainly in the framework of the 1D relativistic fluid approximation, in which solitons are solutions of coupled nonlinear equations for vector and scalar potentials with appropriate boundary conditions. Envelope solitons of circularly polarized EM waves in a cold plasma have been tackled by Kozlov et al. [9], who adopted a quasineutral approximation to prove the existence of small-amplitude localized solutions in the form of drifting solitons. A subsequent numerical investigation focused on solitons with relativistic amplitude, for which charge separation in the plasma is substantial [9]. The exact one-dimensional nonlinear solutions of the relativistic cold plasma equations which represent drifting envelope solitons of circularly polarized light waves were treated by Kaw et al. [18]. This soliton pulse can be interpreted as a light wave which is trapped in a plasma wave generated by itself. A relativistic EM soliton solution with zero group velocity was obtained within a one-dimensional cold plasma model without using the envelope approximation in [19]. In a weakly relativistic approximation, one-dimensional solitary waves in cold plasmas were investigated by means of a perturbation technique by Kuehl and Zhang [20], who found that solitary waves have amplitudes

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which are allowed to take discrete values only. The existence and dynamics of bright and dark solitons and the influence of ion motion on relativistic solitons was investigated by Farina et al. [10,21,22]. The propagation of weakly relativistic circularly polarized EM pulses in a warm plasma in the form of bright and dark solitons was discussed by Poornakala et al. [23], who found different parameter regions of existence for different types of solitons. The 1D relativistic EM solitons in an electron–ion plasma of arbitrary temperature have been investigated in [24–26]. The problem of existence of single peak as well as multiple peak solitary structures and their stability, mutual interaction and propagation in the inhomogeneous cold plasma was tackled by Saxena et al. [27]. The stability of 1D relativistic solitons was treated in [28,29]. We must note that all of the above investigations refer to unmagnetized plasmas.

The generation of intense magnetic field during laser plasma interactions has been the subject of keen attention since it affects the dynamics of the laser pulse as well as the plasma background. The fast ignition scheme for inertial confinement fusion has generated great interest in the study of magnetic fields in laser plasma interaction, and of their effects on the pulse propagation [30]. An extremely intense magnetic field (up to hundreds of MG) has been observed by experimental measurements in the interaction of short laser pulses with a plasmas [31–34]. One particular phenomenon, the inverse Faraday effect (IFE) has been cited as an efficient source of intense magnetic fields in laser plasma interaction, where the propagation of circularly polarized EM waves in plasmas induces an axial magnetic field along the direction of propagation. An axial magnetic field amounting to some MG from IFE has been measured in numerical experiment [35]. Throughout the text, the strength of the magnetic field is represented by the ratio $\Omega = \omega_c/\omega_{pe}$, where $\omega_c$ is the electron cyclotron frequency and $\omega_{pe}$ is the plasma frequency. As a few examples, this ratio can takes the values $\Omega = 0.1$ [35], $\Omega = 1.5$ [31], $\Omega = 0.5$ and $\Omega = 2$ [34] in laser matter interaction experiments.

Since the major part of existing investigations dedicated to relativistic solitons have been performed in unmagnetized plasma systems, it is tempting to incorporate a magnetic field into a model for the interaction of circularly polarized laser pulse with plasmas, and study how the magnetic field affect the soliton structure. Shukla and Stenflo [36], have reported stationary soliton structures for the electromagnetic fields in the framework of a slowly varying approximation. The existence of large amplitude localized electromagnetic pulses in magnetized plasma was considered by Nagesha et al. [37], who have adopted a slowly varying approximation and neglected perturbations in the longitudinal component of the electron momentum, to find stationary solutions for circularly polarized EM waves in cold magnetized plasmas. Near-sonic envelope EM waves were inspected by Rao [38]. It is found that for a given magnetic field strength, left-hand circularly polarized (LCP) waves have larger amplitude than right-hand circularly polarized (RCP) waves, and a similar result was obtained for the associated low frequency density perturbation. The existence of standing one-dimensional relativistic solitons in a cold magnetized plasma and the effects of a magnetic field on soliton stability was treated by Farina et al. [39]. It is found that the frequency interval of stability strongly depends both on the magnitude and on the orientation of the magnetic field. Moreover the maximum field amplitude characterizing the corresponding soliton profiles depends on the externally exposed magnetic field.

In this work, we start by demonstrating the existence of localized EM modes (bright and dark-type envelope solitons) in magnetized plasmas. We employ a multi-scale perturbation method to treat the problem, in which most of the simplifying assumptions and approximations included in earlier papers have been abandoned. Furthermore, we have included the parallel component in the expression for the electron momentum, which has been omitted in earlier works. We proceed in our investigation of the evolution of solitons in magnetized plasma by addressing the stability of these solutions.

The outline of this Letter is as follows. The governing equations are introduced in Section 2. In Section 3, a perturbation technique is employed, leading to a nonlinear Schrödinger equation for the amplitude of the vector potential. The fundamental information for a modulational instability analysis is provided in Section 4. In Section 5, the occurrence of envelope solitons is addressed. The parametric investigation for either LCP or RCP waves is presented in Section 6, and the results of numerical simulation have been presented in Section 7. Finally, we summarize our results in Section 8.

## 2. Governing equations

We consider the propagation of a circularly polarized (CP) electromagnetic wave in a cold plasma embedded in a uniform magnetic field $\mathbf{B}_0$. The one-dimensional (1D) approximation in wave propagation is adopted throughout this text, implying that the radiation spot size is large compared to the plasma wavelength, and thus all field and plasma quantities depend only to one coordinate variable. Here we consider EM field propagation along the $x$ axis, so $\partial/\partial y = \partial/\partial z = 0$. In terms of the scalar and vector potentials, here denoted by $\phi$ and $\mathbf{A}$ respectively, the governing fluid and Maxwell equations in the limit in which ions are assumed to be stationary can be written in the form [39]

$$\frac{\partial^2 \mathbf{A}}{\partial x^2} - \frac{\partial^2 \mathbf{A}}{\partial t^2} = \frac{\partial^2 \phi}{\partial t \partial x}\hat{x} + \frac{n}{\gamma}\mathbf{P}, \tag{1}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n - 1, \tag{2}$$

$$\frac{\partial n}{\partial t} + \frac{\partial (n v_x)}{\partial x} = 0, \tag{3}$$

$$\frac{\partial (\mathbf{P} - \mathbf{A})}{\partial t} = \frac{\partial (\phi - \gamma)}{\partial x}\hat{x} + \mathbf{V} \times \left(\frac{\partial}{\partial x}\hat{x}\right) \times (\mathbf{P} - \mathbf{A}) - \mathbf{V} \times \mathbf{B}_0, \tag{4}$$

$$\gamma = \sqrt{1 + P^2}, \tag{5}$$

where $\mathbf{V} = \mathbf{P}/\gamma$ is the fluid velocity ($\mathbf{P}$ is the electron momentum), $\gamma$ is the relativistic factor, $\mathbf{A}$ and $\phi$ are the vector and scalar potentials, respectively. We have normalized the scalar and vector potentials by $mc^2/e$, $\mathbf{E}$ by $mc\omega_{pe}/e$, $\mathbf{B}$ by $m\omega_{pe}/e$ (where $\omega_{p0}^2 = n_0 e^2/\epsilon_0 m_e$; $n_0$ is the ambient plasma density), the momentum by $mc$, the density by the ambient plasma density $n_0$, the electron velocity by the light velocity $c$; furthermore, the length is normalized by the skin length $c/\omega_{p0}$, and time is scaled by the plasma period (inverse plasma frequency) ($\omega_{p0}^{-1}$).

We consider electromagnetic pulse wave propagation in the direction parallel to the ambient magnetic field, i.e. along the $x$ axis, by assuming $\mathbf{B}_0 = \Omega\hat{x}$, where $\Omega$ is the cyclotron frequency normalized by the ambient plasma frequency $\omega_{pe}$. The state of polarization of an EM wave propagating along the magnetic field in homogeneous plasma is left unchanged during wave propagation [40]. For a circularly polarized EM pulse, the vector potential and electron momentum can be expressed as

$$\mathbf{A} = A(x,t)(\hat{y} + i\alpha\hat{z}), \tag{6}$$

$$\mathbf{P} = p(x,t)(\hat{y} + i\alpha\hat{z}) + \gamma u(x,t)\hat{x}, \tag{7}$$

where $p(x,t)$ is the transverse component of the electron momentum, $u(x,t)$ is the longitudinal component of the electron velocity and $\alpha = 1$ ($\alpha = -1$) for left- (right-) hand circular polarization of the EM pulse. Inserting Eqs. (6) and (7) into Eqs. (1)–(5) we obtain