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The gravitational resolving power of global seismic networks in the 0.1–10 Hz band

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ABSTRACT

Among the first attempts to detect gravitational waves, the seismic approach pre-dates the digital era. Major advances in computational power, seismic instrumentation and in the knowledge of seismic noise suggest to reappraise its potential. Using the whole earth as a detector, with the thousands of digital seismometers of seismic global networks as a single phased array, more than two decades of continuous seismic noise data are available and can be readily sifted at the only cost of (a pretty gigantic) computation. Using a subset of data, we show that absolute strains $h \lesssim 10^{-17}$ on burst gravitational pulses and $h \lesssim 10^{-21}$ on periodic signals may be feasibly resolved in the frequency range 0.1–10 Hz, only marginally covered by current advanced LIGO and future eLISA. However, theoretical predictions for the largest cosmic gravitational emissions at these frequencies are a few orders of magnitude lower.

1. Introduction

The Gravitational Waves (from now on *GW*), i.e., the oscillatory disturbances of the space–time induced by the unbalanced motion of masses measurable on or around the Earth are expected to be exceedingly small [1,8,31]. A basic idea [5,13] alternative to measure directly the geometrical perturbation – as is pursued in gravitational interferometers – is to infer it from the elastic strains that *GW* induce on a material continuum.

1.1. Seismic disturbance produced by gravitational waves

Choosing a rectilinear coordinate system, which is justified by the relative smallness of the earth's mass, and assuming local elastic isotropy, the *GW* induce a displacement η

$$\frac{\partial}{\partial t}\left(\rho\frac{\partial \eta_i}{\partial t}\right) = \frac{\partial}{\partial x_j}\left[\lambda\delta_{ij}\epsilon_{kk} + 2\frac{\partial \mu}{\partial x_j}\left(\epsilon_{ij} + \frac{1}{2}h_{ij}\right) + 2\mu\frac{\partial \epsilon_{ij}}{\partial x_j}\right] \qquad (1)$$

where ϵ_{ij} is the strain tensor, h_{ij} is the gravitational perturbation, ρ is the density, and λ, μ are the Lamé constants. It is clear from equation (1) that *GW* generate elastic strains only at rigidity discontinuities, where $\partial\mu/\partial x \neq 0$. In most rigid bodies (e.g., bars, resonating spheres, etc.) this is realized at the surface. The extreme mismatch between the velocities of light and elastic waves makes very small the amplitude of the elastic strain waves – from now on *GEW* – excited by *GW*.

The largest available elastic body on earth is the planet itself. The gravitational displacement η_i can be measured by the moving mass of a seismometer, since at frequencies f higher than the instrumental eigenfrequency f_0 it approximates a free inertial mass, which can be used as an absolute reference for the strains induced on the environment. Given that $\mu = \rho v_s^2$, with v_s the velocity of shear elastic waves, there exist two major rigidity discontinuities in the earth, which are located at the earth surface, and at the Core–Mantle Boundary (from now on *CMB*), where there is a transition from the solid mantle to the liquid outer core.

As apparent in Table 1, the largest of these discontinuities is at the *CMB* [e.g. [14]], where μ drops from 2900 kbar to 0 while at the surface it goes from 0 to 270 kbar. Hence, the related *GEW* initial amplitude is larger by approximately one order of magnitude at the *CMB* than at the surface, partly compensated by a geometric cross section which is smaller by approximately a factor of 4.

The *GEW* consist primarily of acoustic-like pressure P waves and by shear S waves traveling radially through the Earth's mantle and crust in both directions. The interference of the upgoing and downgoing waves combine to induce standing and guided waves according to the medium layering. At 1–50 mHz frequencies, the earth global spheroidal and toroidal normal modes [cf. [11]] are continuously excited by the oceanic noise [30]. In the frequency range we consider, since at the time-scale of elastic waves the whole Earth is almost simultaneously excited by the *GW*, it is feasible as a first approximation to use a 1-D flat earth model

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Table 1
Earth's mantle elastic and anelastic parameters compiled by merging the main reference models [14,19,28].

Layer thickness (km)	v_P (km/s)	v_S (km/s)	Density (g/cm^3)	Q_P	Q_S
0.4	1.6	0.8	2.0	100	50
30	5.8	3.3	2.7	200	80
20	6.1	3.5	2.90	200	80
70	8.5	4.9	3.30	200	80
630	9.0	5.2	4.50	900	450
2100	12.0	7.0	5.50	900	450

[cf. [13]]. Using this in a Haskell–Thomson approach [16] together with the elastic wave velocities, density and Q reported in Table 1 – compiled merging the state of the art earth models (cf. the caption of Table 1) – we calculate the response of the earth to a *GW* excitation at the surface and *CMB*. At the surface, where seismic measurements are taken, some amplification with a factor $F \lesssim 8$ is obtained (see Fig. 1) on the vertical component (P waves), and $F \lesssim 6$ on the horizontal component (S waves). Note that this amplification will also depend on the near surface stratigraphy of the specific site, which might possibly give large amplitudes (cf. [18]), but the same will occur for seismic noise, thus canceling any advantage for *GEW* detection. In conclusion, one may expect a modest amplification for the *GEW* from the Earth's response in the band 0.1–10 Hz, smaller than one order of magnitude, and we will not explicitly consider it in the following.

Let us estimate the amplitude of the *GEW* strains that can be feasibly resolved above seismic noise in this frequency band by using the seismic stations in operation worldwide as a single gravitational antenna. These stations mostly comply to the standards of IRIS – the Incorporated Research Institutions for Seismology – and FDSN – Federation of Digital Seismographic Networks – granting the online availability of the continuous waveforms for $\sim 2 \times 10^3$ instruments around the world. Such instruments are triaxial broad-band 0.005–10 Hz digital 32 bit force balance seismometers, usually Streckeisen STS1/STS2, Guralp 3T or Trillium 240, mostly sampled at 20 Hz. All seismic station sites have been carefully pre-selected for their particularly low background noise, and each station consists of thermostatized seismometer cases placed on concrete bases built on a rock outcrop and located inside a specialized building. Some instruments are placed in galleries, abandoned mines or deep boreholes to achieve an optimal insulation.

2. The main features of seismic noise

Seismic noise, produced by both natural and anthropic causes, is present everywhere on earth. Phenomenologically, its main feature lies in a stochastic interference origin, i.e., it is the result of the interaction of waves from a variety of direct and Huygens sources. An empirical envelope of the "minimum" seismic noise, simply estimated as the lower bound of the recorded values over a moderately large record, is summarized in the NLNM model [23]. Statistically, a better defined estimate of "low noise" is its most frequent value, identified according to the *statistical mode* of the Gumbel I extreme value statistics. This was estimated on a one year long experimental analysis of the MEDNET stations and parameterized in the Statistical Low Noise Model – SLNM [9]. At frequencies $f > 10^{-3}$ Hz, both NLNM and SLNM show a roughly flat power spectral density (from now on *PSD*) in acceleration $PSD(\ddot{x})$, with a slightly higher plateau level at frequencies >1 Hz for the additional contribution of the anthropic noise sources (Fig. 2). Notable deviations from such a white noise-like behavior are the two broad spectral humps around 0.2 Hz and 0.07 Hz.

Since we are essentially interested in identifying a specific signal above noise, the most frequent low noise value represents a more reliable reference than its empirical lower bound. We therefore refer to SLNM rather than NLNM, according to which the noise level is approximated for both the vertical and the horizontal components by [cf. Fig. 2 and Table 1 of [9]]

$$PSD(\ddot{x}) \sim const \sim 10^{-16}\ \mathrm{m^2\,s^{-4}/Hz}, \tag{2}$$

with a corresponding power spectral density in displacement [cf. [6]] of $PSD(x) \sim 10^{-20} f^{-4}\ \mathrm{m^2/Hz}$, i.e., power law decreasing with frequency from $\sim 10^{-16}\ \mathrm{m^2/Hz}$ at 0.1 Hz to $\sim 10^{-24}\ \mathrm{m^2/Hz}$ at 10 Hz (see Fig. 2). The constant power spectral density in acceleration stands for seismic noise being the result of the stochastic sum of several uncorrelated processes, an issue corroborated by the experimental evidence that in the range 10–10^3 s noise is also stationary and Gaussian [26].

A further main feature of seismic noise is its strong site dependence, with sites not specifically selected for quietness – and in particular those close to coasts or industrial and urban areas – showing noise power spectral densities larger by several orders of magnitude than SLNM. For example, the noise measured at the VIRGO gravitational interferometer site has a power spectral density $PSD(\ddot{x}) \sim 10^{-11}\ \mathrm{m^2\,s^{-4}/Hz}$ [7]. Most theoretical analyses of seismic noise have been formulated in terms of the elastic Green's function [3,20]. While well suited to evaluate the response of an

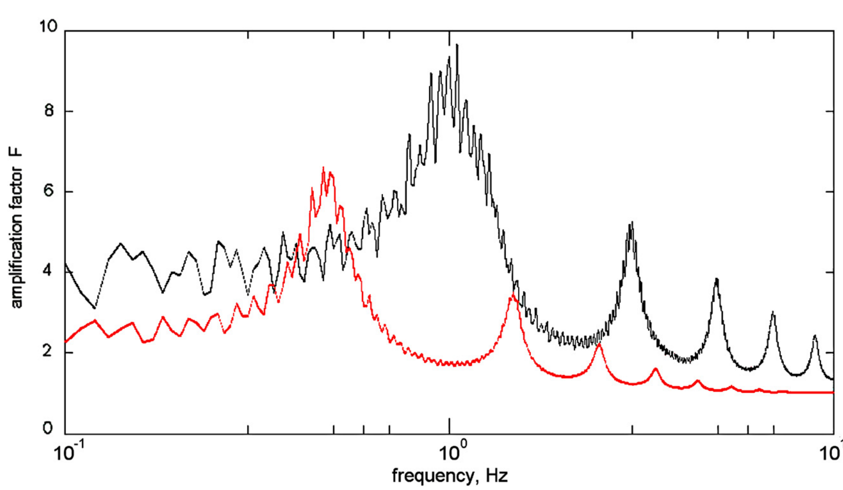

Fig. 1. The response of the Earth mantle and crust, represented by the model in Table 1, in terms of vertical (black) and horizontal (red) displacements respectively due to P and SH elastic waves generated by *GW* at the earth's surface and core–mantle boundary. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)