Physics Letters A 380 (2016) 1542–1546

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Anti-dynamical Casimir effect with an ensemble of qubits

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ARTICLE INFO

Article history:
Received 8 January 2016
Received in revised form 18 February 2016
Accepted 19 February 2016
Available online 24 February 2016
Communicated by P.R. Holland

Keywords:
Nonstationary circuit QED
Anti-dynamical Casimir effect
Rabi Hamiltonian
Coherent photon annihilation
Effective Hamiltonian

ABSTRACT

We consider the interaction between a single cavity mode and $N \gg 1$ identical qubits, assuming that any system parameter can be rapidly modulated *in situ* by external bias. It is shown that, for the qubits initially in the ground states, three photons can be coherently annihilated in the dispersive regime for harmonic modulation with frequency $3\omega_0 - \Omega_0$, where ω_0 (Ω_0) is the bare cavity (qubit) frequency. This phenomenon can be called "Anti-dynamical Casimir effect", since a pair of excitations is destroyed without dissipation due to the external modulation. For the initial vacuum cavity state, three qubit excitations can also be annihilated for the modulation frequency $3\Omega_0 - \omega_0$.

1. Introduction

"Dynamical Casimir effect" (DCE) is a broad term used nowadays to denote a group of phenomena characterized by creation of quanta from some initial state (usually vacuum) of some field due to time-dependent variations of the geometry or the material properties of a macroscopic or mesoscopic system (see [1–5] for recent reviews). The stricter term "*Cavity DCE*" [6] was coined to represent the process of photon generation from the electromagnetic (EM) vacuum and other initial states inside some cavity (resonator) due to fast motion of a wall or the time-modulation of material properties of the boundary or the intra-cavity medium (e.g., dielectric permittivity or conductivity) [7,8]. Traditionally DCE has been studied from the macroscopic viewpoint [1,4,7,9–15], according to which the changing medium or moving walls are effectively described as time-dependent dielectric permittivity or boundary conditions for the field, so there is no need to take into account the internal degrees of freedom of the subsystems that constitute the boundary/medium.

Recently the analogs of DCE have been observed experimentally in the single-mirror [16] and cavity [17] configurations. These experiments were implemented in the solid-state architecture known as "circuit Quantum Electrodynamics" (circuit QED) [4,18,19]. The modulation of the boundary conditions for the EM field inside superconducting coplanar waveguide was achieved by threading time-dependent magnetic flux through a single or an array of SQUIDs (superconductive quantum interference devices) composed of Josephson junctions.

Josephson junctions are also the main constituent of the superconducting artificial atoms (AA) [20–22] that can strongly couple to the resonator field via the dipole interaction [23,24]. As the properties of the artificial atoms can be controlled *in situ* via electric/magnetic fields, it is natural to ask whether one could implement the DCE with a single AA or an array of such atoms, when the atomic internal degrees of freedom play essential role and cannot be replaced by effective time-dependent boundary conditions. This question has been addressed in series of theoretical papers over the last decade [25–32], which indicate that DCE could indeed be implemented with a single two-level atom (also called *qubit*), while the use of atomic ensemble with N noninteracting qubits provides N-fold increase in the photon generation rate [33]. However, the atomic internal degrees of freedom modify the dynamics of DCE – the field becomes entangled with the qubits and the photon generation saturates due to intrinsic Kerr nonlinearity originating from the dispersive light-matter interaction. On the other hand, the atom-field entanglement could be tailored for new applications, e.g., indirect monitoring of the cavity field state [5,6,34–38], generation of multipartite quantum correlations [39,40] and simulation of relativistic motion [41].

The realization of DCE by means of microscopic objects offers another novelty: since the atom-field interaction alters the spectrum of the composite system, new transitions between the bare

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http://dx.doi.org/10.1016/j.physleta.2016.02.031
0375-9601/

eigenstates (dressed states) can be driven via weak resonant modulation of the cavity or atomic parameters [25,26,34,36,37,42,43]. One such transition has been recently discovered in [33] and called attention for coherently destroying photons instead of creating [44]. Thus the system response to harmonic perturbation consists in transferring the energy from the system to the external agent – an effect opposite to the standard DCE, or "*Anti-DCE*". Similar mechanism of energy transfer from the signal and idler beams to the pump beam is known under the name "coherent attenuation" in Nonlinear Optics [45]. Anti-DCE was investigated theoretically for the single-qubit setup and two different dissipation models in [32]. It was shown that Anti-DCE has a rather small transition rate for the currently available parameters and requires the qubit be prepared in the ground state and be detuned from the cavity frequency. Moreover, only two system excitations can be annihilated for a single-tone modulation: destruction of three photons is accompanied by the qubit excitation (more than two excitations can be annihilated if one employs multi-tone modulations).

In this letter we extend the analysis of Anti-DCE to the ensemble of N identical qubits, such as array of superconducting artificial atoms [46] or a cloud of cold polar molecules trapped above the waveguide resonator [47,48]. We consider the harmonic modulation of any system parameter and derive the effective Hamiltonian that governs the dynamics. It is shown analytically and numerically that the Anti-DCE behavior for $N \gg 1$ is analogous to the one found for $N = 1$ [32], with destruction of three photons complemented by creation of one collective atomic excitation, though the associated transition rate undergoes a collective enhancement. Moreover, for a different modulation frequency one could also coherently annihilate three collective atomic excitations if the cavity was initially in the vacuum state ("matter Anti-DCE"). These phenomena take place for stationary qubits too, provided the cavity frequency undergoes external modulation due to time-varying macroscopic boundary conditions.

2. Mathematical formalism

We consider a set of N identical noninteracting qubits with transition frequencies Ω confined in a cavity or trapped above a superconducting waveguide resonator. When the qubits are nearly resonant with the cavity mode of frequency ω we can use the single-mode approximation and write for the total Hamiltonian (we set $\hbar = 1$)

$$\hat{H} = \omega\hat{n} + \sum_{l=1}^{N}\left[\frac{\Omega}{2}\hat{\sigma}_z^{(l)} + g(\hat{a}+\hat{a}^\dagger)(\hat{\sigma}_+^{(l)} + \hat{\sigma}_-^{(l)})\right] + i\chi(\hat{a}^{\dagger 2} - \hat{a}^2). \tag{1}$$

Here $\hat{a}$ ($\hat{a}^\dagger$) is the cavity field annihilation (creation) operator and $\hat{n} = \hat{a}^\dagger\hat{a}$ is the photon number operator. The qubit operators are $\hat{\sigma}_-^{(l)} = |g^{(l)}\rangle\langle e^{(l)}|$, $\hat{\sigma}_+^{(l)} = |e^{(l)}\rangle\langle g^{(l)}|$ and $\hat{\sigma}_z^{(l)} = |e^{(l)}\rangle\langle e^{(l)}| - |g^{(l)}\rangle\langle g^{(l)}|$, where $|g^{(l)}\rangle$ and $|e^{(l)}\rangle$ denote the ground and excited states of the l-th qubit, respectively.

Parameter g stands for the interaction strength between the cavity field and a single qubit, so the term $g(\hat{a}+\hat{a}^\dagger)(\hat{\sigma}_+^{(l)} + \hat{\sigma}_-^{(l)})$ describes the standard dipole interaction [49]. In this work we do not neglect the counter-rotating terms ($\hat{a}\hat{\sigma}_-^{(l)} + \hat{a}^\dagger\hat{\sigma}_+^{(l)}$), since they are responsible for the Anti-DCE. The "squeezing coefficient" χ is included for the sake of generality and is not essential for the main findings of this work. Its time-independent part may arise due to the terms proportional to the square of the vector potential, which appear naturally when one uses the minimal-coupling Hamiltonian and the dipole approximation of the first-order or higher [33,49–51]. The time-dependent part of χ is related to some form of parametric amplification process [4]; in the context of nonstationary phenomena one can show that for external time-modulation of the cavity frequency it reads (in the simplest case) $\chi = (4\omega)^{-1}d\omega/dt$ [52,53].

In this paper we assume that all the parameters of the Hamiltonian can be modulated by external bias (simultaneously or one at a time) as

$$X = X_0 + \varepsilon_X \sin(\eta t + \phi_X)\ ,\ X = \{\omega, \Omega, g, \chi\}\,. \tag{2}$$

Here X_0 is the bare value and $\varepsilon_X \geq 0$ is the modulation depth of X. The modulation frequency is η, assumed to be of the order $\sim 2\omega_0$, and ϕ_X is the phase associated to the modulation of the parameter X.

The case $N = 1$ was studied thoroughly in [32], so here we consider the opposite scenario of ensemble of qubits, $N \gg 1$ [31,33]. First we define the collective matter operators via the Holstein–Primakoff transformation [54]

$$\sum_{l=1}^{N}\hat{\sigma}_+^{(l)} = \hat{b}^\dagger(N - \hat{b}^\dagger\hat{b})^{1/2}\ ,\ \sum_{l=1}^{N}\hat{\sigma}_-^{(l)} = (N - \hat{b}^\dagger\hat{b})^{1/2}\hat{b}$$
$$\sum_{l=1}^{N}\hat{\sigma}_z^{(l)} = 2\hat{b}^\dagger\hat{b} - N\,, \tag{3}$$

where the ladder operators $\hat{b}$ and $\hat{b}^\dagger$ satisfy the bosonic commutation relation $[\hat{b}, \hat{b}^\dagger] = 1$. To the first order in $\hat{b}^\dagger\hat{b}/N$ the Hamiltonian (1) becomes

$$\hat{H} = \omega_0\hat{n} + \Omega\hat{b}^\dagger\hat{b} + \tilde{g}(\hat{a}+\hat{a}^\dagger)(\hat{b}+\hat{b}^\dagger) + i\chi(\hat{a}^{\dagger 2} - \hat{a}^2)$$
$$- \frac{\tilde{g}}{2N}(\hat{a}+\hat{a}^\dagger)(\hat{b}^{\dagger 2}\hat{b} + \hat{b}^\dagger\hat{b}^2)\,, \tag{4}$$

where we defined the collective coupling strength[1] $\tilde{g} \equiv \sqrt{N}g$, so that $\tilde{g}_0 = \sqrt{N}g_0$ and $\tilde{\varepsilon}_g = \sqrt{N}\varepsilon_g$ (we consider $g_0 \geq 0$ without loss of generality). The Hamiltonian (4) holds provided the inequality $|b^\dagger b|_{max} \ll N$ is satisfied, where $|b^\dagger b|_{max}$ is the maximum number of qubits' excitations.

Following the method developed in [31] we write the solution in the Heisenberg picture as

$$\hat{a}(t) = \frac{e^{-it\Delta_+/2}}{\beta}\left\{[\beta_+\hat{A}_h(t) + \tilde{g}_0\hat{B}_h(t)]e^{-it\beta/2}\right.$$
$$\left. + [\beta_-\hat{A}_h(t) - \tilde{g}_0\hat{B}_h(t)]e^{it\beta/2}\right\} \tag{5}$$

$$\hat{b}(t) = \frac{e^{-it\Delta_+/2}}{\beta}\left\{[\beta_-\hat{B}_h(t) + \tilde{g}_0\hat{A}_h(t)]e^{-it\beta/2}\right.$$
$$\left. + [\beta_+\hat{B}_h(t) - \tilde{g}_0\hat{A}_h(t)]e^{it\beta/2}\right\}\,, \tag{6}$$

where $\Delta_+ \equiv \omega_0 + \Omega_0$, $\beta \equiv \sqrt{\Delta_-^2 + 4\tilde{g}_0^2}$ and $\beta_\pm \equiv (\beta \pm \Delta_-)/2$. $\Delta_- \equiv \omega_0 - \Omega_0$ is the detuning between the bare values of the cavity and qubit frequencies.

Next we introduce the new operators $\hat{A}$ and $\hat{B}$ via the implicit relations

$$\hat{A}_h = e^{it\tilde{\delta}_+}(\hat{A}e^{it\delta_\chi} + i\mathcal{F}_{AB}\hat{B})e^{i\mathcal{F}_A}$$
$$\hat{B}_h = e^{it\tilde{\delta}_+}(\hat{B} + i\mathcal{F}_{AB}^*\hat{A}e^{it\delta_\chi})e^{i\mathcal{F}_B}\,, \tag{7}$$

where we defined the resonance shifts $\tilde{\delta}_+ = \tilde{g}_0^2/\Delta_+$ and $\delta_\chi = 4\chi_0^2/\Delta_+$. The small time-dependent c-number functions are

[1] In this paper the tilde over a c-number denotes the collective N-qubits parameter, whereas the respective quantity without the tilde is the single-qubit parameter.