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Aspiration dynamics and the sustainability of resources in the public goods dilemma

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ABSTRACT

How to exploit public non-renewable resources is a public goods dilemma. Individuals can choose to limit the depletion in order to use the resource for a longer time or consume more goods to benefit themselves. When the resource is used up, there is no benefit for the future generations any more, thus the evolutionary process ends. Here we investigate what mechanisms can extend the use of resources in the framework of evolutionary game theory under two updating rules based on imitation and aspiration, respectively. Compared with imitation process, aspiration dynamics may prolong the sustainable time of a public resource.

1. Introduction

It is so ubiquitous to misuse public non-renewable resources that it becomes a hot topic in the international forum nowadays. One of the characteristics of public resources is that the use by one reduces the quantity or quality available to others. These resources include fossil power, groundwater basins and atmosphere. Obviously, the overexploitation of resources today has a high cost on the welfare of future generations [1–5]. Benefitting future generations is important to the survival of genes, families, organizations, nations and the global ecosystem. Yet it is challenging, as it requires that individuals in this generation limit themselves in using those resources and make sacrifices today.

The public resource dilemma is highly focused on in today's global society, because it determines how long we will hold those necessary resources for the sustainable development. An excellent example of such a dilemma concerns the mitigation of the effects of global warming. It has been described as one of the biggest public goods dilemmas that all of us have to face and one that we cannot afford to lose [4–9]. Indeed, given that the atmosphere of our planet is an indivisible good accessible by all, individuals, regions, or nations can choose to reduce the emissions of greenhouse gases or deforestation (cooperation) or not (defection). The ideal case under cooperation is to use the non-renewable resource for a long time. However, individuals or organizations may opt to be free riders in such global public goods games, hoping to benefit from the efforts of others while choosing not to make any effort themselves.

In general, whether or not devoting efforts targeting at the mitigation of future losses, i.e., the collective risk dilemma, depends on how likely the disaster happens. Such a dilemma is beautifully captured in simple and elegant experiments [7–11] and models [12–16]. Those experiments and models make use of a repeated game to study whether the target can be met given a time period [5,17–19]. However, it is seldom addressed how the time until which the resource is run out of is influenced. In this paper, we establish an evolutionary game model to capture the sustainable time of public goods. We study the sustainable time, and focus on how the sustainable time is affected by human strategy updating. Here we consider two updating rules: imitation and self-learning based on one's own aspiration [20–23]. We are proposing a novel theoretical question: how fast does an evolutionary process end? Typically, game theoretical models rooted in population genetics focus on the fixation time when the population size is finite. But the evolution process can still be well defined when the fixation event happens. Even though the population configuration does not change, everyone still learns each other based on the updating rule. Here, however, when the resource is used up, there is no consumption behavior any more, thus the evolutionary process really ends. We utilize methods of statistical physics to analytically study

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the sustainable time of public goods, and find that when we adopt a self-regulated method to update our strategy we are likely to use the resource for a longer time than the case in which we use an imitation process. Further, the sustainable time is the shortest for the intermediate aspiration level, and strong selection shortens the evolutionary time of the public resource dilemma.

2. Model

2.1. The dilemma of resource consumption

We consider evolutionary game dynamics with two strategies in a finite well-mixed population of size N. In this game, we assume that initially the population owns public goods of amount P. A focal player can be of type A or B. Individuals of type A and B consume the public goods and acquire the payoffs at the amount of a and b, respectively ($0 < a \le b$). Thus strategy A is cooperation and strategy B is defection. If there are i A players in the population, thus $N-i$ individuals of type B, there remains $P - ai - b(N-i)$ amount of public goods after one time step.

This game theoretic framework is a simplification compared to a real-world cooperation dilemma. This simplification, however, captures key elements of the sustainable challenge facing our world [3]: the game is non-zero sum, with cooperation today creating greater benefits for the future.

2.2. Updating rules

We adopt two different updating rules: imitation which is based on individuals' information about their opponents, and aspiration which is based on individuals' information about themselves.

In imitation dynamics, an individual, namely F, is selected uniformly at random from the entire population of size N. Then, we randomly choose another individual, namely G. Subsequently, F adopts G's strategy with probability $1/\{1+\exp[-\omega(\pi_G - \pi_F)]\}$ [24–29]. Here π_x is the payoff of individual x. ω denotes the imitation intensity, measuring the dependence of decision-making on the payoff difference. For $\omega \to 0$, individual F imitates the strategy of G almost randomly, which is referred as "weak selection." For $\omega \to \infty$, a more successful player is very likely imitated, which is referred as "strong selection."

The state of the system is denoted as (i, g) where i is the number of individuals using strategy A and g is the amount of resources left at time t. Thus if at time t, the system is at state (i, g), then at the next time three events are possible: state (i, g) can be changed to $(i+1, g-[ai+(N-i)b])$ with probability $T^+(i)$, to $(i-1, g-[ai+(N-i)b])$ with probability $T^-(i)$, or to $(i, g-[ai+(N-i)b])$ with probability $T^0(i)$. All other transitions cannot occur. The transition probabilities are given by [27]:

$$T^+(i) = \frac{N-i}{N}\frac{i}{N}\frac{1}{1+\exp[-\omega(a-b)]},$$
$$T^-(i) = \frac{i}{N}\frac{N-i}{N}\frac{1}{1+\exp[-\omega(b-a)]}, \quad \text{and}$$
$$T^0(i) = 1 - T^+(i) - T^-(i). \tag{1}$$

For aspiration-driven updating, players are likely to switch strategies if the aspiration level is not met, where the level of aspiration is an intrinsic property of the focal individual [30–32]. For a randomly chosen individual, it compares its payoff π_x with the aspiration level α to decide whether switching its strategy or not. The probability to switch is $1/\{1+\exp[-\omega(\alpha - \pi_x)]\}$ [22,23]. Here $\omega > 0$ is the selection intensity. The level of aspiration provides a global benchmark of tolerance or dissatisfaction in the population. The transition probabilities are given by

$$T^+(i) = \frac{N-i}{N\{1+\exp[-\omega(\alpha-b)]\}},$$
$$T^-(i) = \frac{i}{N\{1+\exp[-\omega(\alpha-a)]\}}, \quad \text{and}$$
$$T^0(i) = 1 - T^+(i) - T^-(i). \tag{2}$$

In both dynamics, novel strategies (apart from the two prescribed strategies) cannot emerge without additional mechanisms such as spontaneous exploration of strategy space (similar to mutation) [27,33–37]. The major difference is that the aspiration-driven updating rule does not require any knowledge about the payoffs of others. Thus aspiration level based dynamics, a form of self-learning, requires less information about an individual's strategic environment than imitation dynamics.

Without resource consumption, compared to imitation (pairwise comparison) dynamics, aspiration dynamics has no absorbing boundaries. Even in a homogeneous population, there is a positive probability that an individual can switch to another strategy owing to the dissatisfaction resulting from payoff–aspiration difference. This facilitates the escape from the states that are absorbing in the pairwise comparison (imitation) process and other Moran-like evolutionary dynamics. However, once the resource is taken into account as in our model, the state has been changed. For both aspiration and imitation processes, there exist absorbing states at which the resource is used up.

2.3. Sustainable time of public resources

Let us denote by $\tau_{i,g}$ the average time such that the system first hits the state where there is no resource starting from state (i, g). The following aim is to get the analytical expression for the sustainable time $\tau_{i,g}$.

Note that $\tau_{i,g}$ is governed by the following Kolmogorov forward equation:

$$\tau_{i,g} = 1 + \tau_{i+1,g-q(i)}T^+(i) + \tau_{i,g-q(i)}T^0(i) + \tau_{i-1,g-q(i)}T^-(i), \tag{3}$$

where $q(i) = ai + b(N-i)$. The boundary condition is $\tau_{j,0} = 0$ where $j \in \{0, 1, \cdots, N\}$.

For $a = b$, the resources are decreased by aN for each time step. This is independent of the updating process. Thus the time to hit the state in which resource is run out of will be g/aN, i.e., $\tau_{i,g} = g/aN$ in this case. What is left is to figure out a method to calculate $\tau_{i,g}$, when $a \ne b$.

Rearranging Eq. (3) and replacing i with $i+1$ lead to

$$\begin{aligned}&\tau_{i+1,g} - \tau_{i+1,g-q(i+1)}\\&\quad = 1 + T^+(i+1)\,[\tau_{i+2,g-q(i+1)} - \tau_{i+1,g-q(i+1)}]\\&\qquad + T^-(i+1)\,[\tau_{i+1,g-q(i+1)} - \tau_{i,g-q(i+1)}].\end{aligned} \tag{4}$$

For $a = b$, based on the above analysis we have $\tau_{i+2,g-q(i+1)} = \tau_{i+1,g-q(i+1)} = [g - q(i+1)]/a$. For $a \approx b$, we expect the solution is continuous thus the difference of those two times should be small, i.e., $\tau_{i+2,g-q(i+1)} - \tau_{i+1,g-q(i+1)} \to 0$. By the same argument, we have that $\tau_{i+1,g-q(i+1)} - \tau_{i,g-q(i+1)} \to 0$. Thus by Eq. (4), we have $\tau_{i,g} - \tau_{i,g-q(i)} \approx 1$. What is noteworthy is that it is true for both imitation and aspiration updating rules. Therefore

$$\underbrace{(\tau_{i+1,g-q(i)} - \tau_{i,g-q(i)})}_{D_1} = \underbrace{\frac{T^-(i)}{T^+(i)}}_{\gamma_i}(\tau_{i,g-q(i)} - \tau_{i-1,g-q(i)}), \tag{5}$$

$$\begin{aligned}&(\tau_{i+2,g-q(i+1)} - \tau_{i+1,g-q(i+1)})\\&\quad = \frac{T^-(i+1)}{T^+(i+1)}\underbrace{(\tau_{i+1,g-q(i+1)} - \tau_{i,g-q(i+1)})}_{D_2}.\end{aligned} \tag{6}$$