Physics Letters A 380 (2016) 1442–1445

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Simple model for fine particle (dust) clouds in plasmas

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ARTICLE INFO

Article history:
Received 11 November 2015
Received in revised form 8 February 2016
Accepted 14 February 2016
Available online 18 February 2016
Communicated by F. Porcelli

Keywords:
Fine particle (dusty) plasma
Structure of dust cloud
Cylindrical symmetry
Spherical symmetry
Microgravity

ABSTRACT

In the cloud of fine particles (dusts) in plasmas, the charge neutrality can be much enhanced due to large charge numbers of fine particles. The required condition is not difficult to satisfy even when their charge density is substantially smaller than electrons or ions. Based on this fact, a simple model of fine particle clouds is proposed and the cloud radius is related to the half-width, the radius where the density of surrounding plasmas drops by half, in cylindrically and spherically symmetric cases under microgravity. When fine particles are gradually introduced with parameters of surrounding plasma especially the half-width being fixed, the size of clouds first increases and then saturates at the value determined by the plasma half-width, giving a possibility to control the size and density of clouds independently.

1. Introduction

Due to mutual strong Coulomb-like interactions and relative easiness to trace individual orbits, systems of fine (dust) particles in plasmas have been the target of intense investigations [1–4]. In particular, the experiments under microgravity performed on the International Space Station (ISS) have elucidated many interesting aspects of the system, being free from the effect of gravity on fine but macroscopic particles.

In order to understand fundamental properties of fine particles, uniform systems with simple geometries and those with as large as possible extensions are naturally desirable. As the former, cylindrical systems such as PK-4 [5] which succeeded highly successful PK-3 Plus [6] may be one of the candidates. Based on the behavior of the net charge density in fine particle clouds, we here give a simple model of fine particle clouds in plasmas with cylindrical and spherical symmetries, hoping the apparatus with the latter symmetry might also be realized in the future.

We consider the system composed of electrons, ions, and fine particles in a weakly ionized plasma where the plasma is generated by dc or rf discharge and the overall flow of plasma is somehow suppressed. As typical values, we assume the plasma density of 10^8–10^9 cm^{-3}, the fine-particle density of $10^4 - 10^5$ cm^{-3}, and typical temperatures of (1–3) eV/k_B, 300 K, and 300 K for electrons, ions, and particles, respectively. We also assume that the gas at the room temperature has the pressure of 10–100 Pa and all components experience frequent collisions with neutral gas atoms and the drift-diffusion equations are applicable [7]. Even if these assumptions are somewhat oversimplified, we expect the results may serve as a basis to take other effects into account.

2. Properties of solutions of drift-diffusion equations

Our analysis is based on the drift-diffusion equations for densities $n_{e,i,p}$ described by diffusion coefficients and mobilities by $D_{e,i,p}$ and $\mu_{e,i,p}$, suffixes e, i, p denoting electrons, ions, and fine particles, respectively:

$$-D_e \Delta n_e - \mu_e \nabla \cdot (n_e \mathbf{E}) = c_g n_e - c_p n_p,$$

$$-D_i \Delta n_i + \mu_i \nabla \cdot n_i (\mathbf{E} - \frac{n_p \mathbf{F}^{id}}{e n_i}) = c_g n_e - c_p n_p,$$

and

$$-D_p \Delta n_p - \mu_p \nabla \cdot n_p \left(\mathbf{E} + \frac{\mathbf{F}^{id}}{(-Qe)} \right) = 0.$$

As shown on the right-hand sides, our plasma (electrons and ions) is assumed to be generated by the impact ionization by electrons with the rate c_g and lost to the surface of a fine particle with the rate c_p (and also to the wall of the apparatus). Three components have different temperatures $T_{e,i,p}$ and the Einstein relations are $D_{e,i}/\mu_{e,i} = k_B T_{e,i}/e$ and $D_p/\mu_p = k_B T_p/Qe$, $-Qe$ being the charge on a particle. The terms with $\mathbf{F}^{id}$ and $(n_p/n_i)\mathbf{F}^{id}$ express the ion drag force on particles and its reaction, respectively. The electric field $\mathbf{E}$ is related to the charge density via the Poisson equation. The above equations are characterized by the scale

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http://dx.doi.org/10.1016/j.physleta.2016.02.016
0375-9601/

length $R_a = (D_a/c_g)^{1/2}$ where D_a is the ambipolar diffusion coefficient $D_a = (\mu_e D_i + \mu_i D_e)/(\mu_e + \mu_i) \approx (T_e/T_i)D_i$.

We consider the case where the system has the cylindrical or spherical symmetry, implicitly assuming that the effect of gravity can be neglected (though our model is applicable also under the gravity, these simple geometries of fine particle clouds are not expected and other considerations are necessary). The above equations then reduce to ordinary differential equations for the functions of the radius. The force by electric field and the ion drag force have only the radial components and, in what follows, we neglect the latter: Their balance is related to the formation of void and will be considered elsewhere.

Without fine particles, we have the quasi charge neutrality $n_e \sim n_i \sim n$ and n satisfies the ambipolar diffusion equation

$$\frac{1}{R}\frac{d}{dR}R\frac{d}{dR}n + \frac{1}{R_a^2}n = 0, \tag{1}$$

R being the radius in the cylindrically symmetric case [8], or

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}n + \frac{1}{R_a^2}n = 0, \tag{2}$$

r being the radius in the spherically symmetric case.

It can be shown that, when we have fine particles of 'appreciable' amount (in the sense given below), the charge neutrality around the axis or the center is much enhanced by their very existence. This behavior is also confirmed by numerical solutions of the drift-diffusion equations in the case of cylindrical symmetry [9]. An example of numerical solutions with cylindrical symmetry is shown in Fig. 1 where main parameters are; particles of radius 1 μm in Ar gas of 40 Pa, $n_e(R=0) = 10^8$ cm^{-3}, $n_p(R=0) = 4\times 10^4$ cm^{-3}, $Q = 9.35\times 10^2$, $k_B T_e = 1$ eV, $T_i = T_p = 300$ K, $R_a = 6.24\times 10^{-1}$ cm, and $c_p n_p(0)/c_g n_e(0) = 0.766$. In the cloud, we observe enhanced charge neutrality, almost flat potential and accordingly flat electron distribution, and much reduced electric field. (A narrow positive peak of the net charge density comes from the difference in the diffusion coefficients of ions and particles: Though being able to compensate particle charges inside of the cloud, ions cannot follow the fast decrease of the latter at the boundary of the cloud, giving such a structure. The electric field and potential, however, are not sensitive to these narrow structures.)

The tendency of flat potential has been noticed in earlier simulations in the case of high particle densities [10–12] but the necessary condition has not been clarified. The condition can be derived by solving the drift-diffusion equations in the form of an expansion around $R = r = 0$.

In the cylindrically symmetric case, we obtain the normalized charge density Δ in the form

$$\Delta \equiv \frac{n_i - n_e - Qn_p}{n_e(0)} = \Delta(R=0)\left[1 + aR'^2 + \dots\right] \tag{3}$$

by expanding $n_e - n_e(0)$ *etc.* with respect to $R' = R/R_a$. Here

$$a \sim \frac{1}{4}\left[A - \frac{T_e/T_i}{\Delta(0)}\left(1 - \frac{c_p}{c_g}\frac{n_p(0)}{n_e(0)}\right)\right] \tag{4}$$

and

$$A \sim R_a^2\left[\frac{e^2 n_i}{\varepsilon_0 k_B T_i} + \frac{Q^2 e^2 n_p}{\varepsilon_0 k_B T_p}\right]_{R=0}. \tag{5}$$

In order for the charge density (3) to behave properly, we need $a \sim \mathcal{O}(1)$ and, since $A \gg 1$, we have $\Delta(0)$ in the form

$$\Delta(0) \sim \frac{T_e/T_i}{A}\left(1 - \frac{c_p}{c_g}\frac{n_p(0)}{n_e(0)}\right) + \mathcal{O}\left(\frac{1}{A^2}\right) + \dots. \tag{6}$$

When $n_p = 0$, (4) and (5) reduce respectively to

$$a \sim \frac{1}{4}\left[A_0 - \frac{T_e/T_i}{\Delta(0)}\right] \tag{7}$$

and

$$A_0 \sim R_a^2\left[\frac{e^2 n_i}{\varepsilon_0 k_B T_i}\right]_{R=0}, \tag{8}$$

reproducing the known result

$$\Delta(0) \sim \frac{T_e/T_i}{A_0} \tag{9}$$

given by the solution of (1) as

$$\frac{n_i - n_e}{n_e(0)} \sim \frac{T_e/T_i}{A_0}\left[1 + \frac{J_1^2(R')}{J_0^2(R')}\right] \sim \frac{T_e/T_i}{A_0}\left[1 + \mathcal{O}(1)R'^2 + \dots\right]. \tag{10}$$

Here J_0 and J_1 are the Bessel functions. Since $A_0 \gg 1$ in usual cases, we have the quasi-charge-neutrality controlled by A_0.

We note that Q is of the order of 10^3–10^4 and therefore A can be much larger than A_0 even if $n_e \gg Qn_p$. As indicated by (6), *we have much enhanced charge neutrality when* $A \gg A_0$. The necessary condition for $A \gg A_0$ is $Q^2 n_p/T_p \gg n_e/T_i$; This is what is meant by 'appreciable' amount of fine particles.

In the spherically symmetric case, we write the solution in terms of $r' = r/R_a$ and obtain

$$\Delta = \Delta(0)\left[1 + a_s r'^2 + \dots\right], \tag{11}$$

where

$$a_s \sim \frac{1}{6}\left[A - \frac{T_e/T_i}{\Delta(0)}\left(1 - \frac{c_p}{c_g}\frac{n_p(0)}{n_e(0)}\right)\right]. \tag{12}$$

We thus have (6) again and therefore much enhanced charge neutrality also in this case. Without fine particles, we have

$$a_s \sim \frac{1}{6}\left[A_0 - \frac{T_e/T_i}{\Delta(0)}\right] \tag{13}$$

which reproduces the result given by the solution of (2),

$$\frac{n_i - n_e}{n_e(0)} \approx \frac{T_e/T_i}{A_0}\left[1 + \frac{j_1^2(r')}{j_0^2(r')}\right] \approx \frac{T_e/T_i}{A_0}\left[1 + \mathcal{O}(1)r'^2 + \dots\right], \tag{14}$$

j_0 and j_1 being the spherical Bessel functions.

3. Model of fine particle (dust) clouds

Let us now construct a model for clouds of fine particles in plasmas. We consider the clouds with the cylindrical or spherical symmetry and denote the radius by R_0 or r_0, respectively. As shown above, the charge neutrality in fine particle clouds is largely enhanced by the existence of fine particles and we have almost flat electrostatic potential. We therefore expect a small electric field at the outer boundary of the clouds as shown in Fig. 1.

Outside of the cloud, we have no fine particles and the distribution is described by (1) in the cylindrical case or by (2) in the spherical cases. Their independent solutions are the Bessel functions of 0-th order, $J_0(R')$ and $N_0(R')$, or the spherical Bessel functions of 0-th order, $j_0(r')$ and $n_0(r')$. When we have no fine particles in the system, the distribution is given by the solution which are regular at the origin ($R = 0$ or $r = 0$), $J_0(R')$ or $j_0(r')$. When we have fine particle clouds, the distribution outside of clouds is still described by (1) or (2) but we have to take linear