Physics Letters A 379 (2015) 2951–2955

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Superradiance transition in graphene

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ARTICLE INFO

Article history:
Received 18 July 2015
Received in revised form 21 August 2015
Accepted 25 August 2015
Available online 29 August 2015
Communicated by P.R. Holland

Keywords:
Superradiance
Graphene
Transfer rate

ABSTRACT

We study theoretically the conditions required for the appearance of a superradiance transition in graphene. The electron properties of graphene are described in the single p_z-orbital tight-binding approximation, corresponding to the two interacting sub-lattices. The corresponding model is reduced to the effective two-level pseudo-spin 1/2 system. For each sub-lattice we introduce the electron transfer rate of escape into a continuum. We demonstrate that, under some conditions, the superradiance occurs, and it corresponds to the maximal quantum coherent escape to the continuum.

1. Introduction

The superradiance transition (ST) was first described by Dicke in 1954 [1], when the ensemble of N non-directly interacting two-level atoms interacted through the self-consistent radiation field in a cavity. In this case, the ST results in a significant enhancement of the spontaneous radiation due to quantum coherent effects. Later it was demonstrated that the quantum coherent effects, similar to the ST, occur in many quantum optical systems, nuclear systems (heavy nuclei decay), nano- and bio-systems [2–7]. The ST usually occurs when the discrete (intrinsic) states of the system interact coherently with the continuum spectra (sinks). Then, an adequate approach for describing the eigenstates and the dynamics of the system can be based on an effective non-Hermitian Hamiltonian for intrinsic states [2–7]. In this case, the eigenenergies of the non-Hermitian Hamiltonian become complex.

Recently, a steady-state superradiant laser with less than one intracavity photon was demonstrated with rubidium-87 atomic dipoles [8]. Large enhancement of Förster resonance energy transfer on graphene platforms was discussed in [9]. In [10], a superradiant plasmonic lasing with a giant gain at the plasmon modes in graphene was theoretically analyzed in a wide THz frequency range.

Qualitatively, the ST occurs when the resonances begin to overlap – the spacing between the resonances becomes of the order of the sum of half-widths of these resonances. With further overlapping of resonances, segregation of the eigenenergies takes place, depending on their decay widths. Namely, the wide superradiance eigenstates provide rapid and coherent decay of the initially populated state in the continuum. The subradient eigenstates, with narrow decay widths, survive for a relatively long time. It is also worth clarifying here that for the ST to occur, not a presence of collective effects (many degrees of freedom) is crucial, but a presence of quantum coherent effects. Indeed, the ST can occur for a single exciton in light-harvesting bio-complexes with relatively small number of light-sensitive molecules, and even in a single two-level system interacting with the continuum. All these effects are described in details in [2–7,11]. (See also references therein.)

In this paper, we determine the conditions at which the ST occurs in a single-layer graphene material. We demonstrate how the ST in this system is related to both the occurrence of an exceptional points (EP), when complex eigenvalues coincide, and to the overlapping of two resonances. Even though, instead of graphene some other semimetals and semiconductors can be used, we believe that, to demonstrate the discussed in this paper effects, a single-layer graphene is the most attractive material. Indeed: (i) This material has two bands (valence and conduction) which allows one to reduce the mathematical model to the relatively simple effective two-level system; (ii) These two bands are represented by two cones touching at the Dirac point (for undopped graphene). The presence of the Dirac point is important for our consideration, as this allows one to analyze the ST both in the

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http://dx.doi.org/10.1016/j.physleta.2015.08.030
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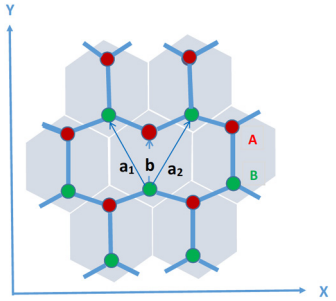

Fig. 1. Graphene two-dimensional lattice, with two atoms, A (red) and B (green), in a unit cell. Primitive unit vectors: $\mathbf{a}_{1,2} = (a/2)(\sqrt{3}, 3)$, $\mathbf{b} = (0, a)$, $a \approx 1.42$ Å. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

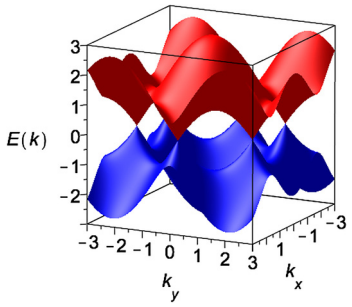

Fig. 2. The band structure of graphene. The valence and conduction bands touch each other in six DPs, where $E(\mathbf{k}) = 0$.

vicinity of the EP (when the standard criterion of resonance overlapping does not work) and in the absence of the EP (when the criterion of resonance overlapping can be used).

We also show that, under some conditions on the parameters and initial population, the maximal efficiency of the electron transfer (ET) into the sink is related to the superradiance state. We compare the ST criterion based on the overlapping of resonances and on the occurrence of the EP. We also demonstrate the influence of noise on the performance of the ET into sinks.

This paper is organized as follows. In Section 2, we introduce the mathematical model. In Section 3, we describe the main properties of the ST in the vicinity of the degeneracy. The ET dynamics, in the vicinity of the EP, is presented in Section 4. In Section 5, the noise-assisted electron transfer in the superradiance regime is described. In the Conclusion (Section 6), we summarize our results and discuss a possible future research in this direction.

2. Description of the model

The structure of a single atomic layer graphene can be described by the honeycomb lattice which consists of two triangular Bravais sub-lattices, represented in Fig. 1 by nonequivalent A (red) and B (green) carbon atoms, which create a unit cell [12,13]. Both sub-lattices have the periodic structures, and are shifted by a vector, $\mathbf{b} = (0, a)$, $a \approx 1.42$ Å, which connects A and B atom in the unit cell. In a carbon atom, six electrons occupy the $1s^2$, $2s^2$, and $2p$ orbitals. From them, four valence electrons are responsible for structural and electronic properties. One of the valence electrons of each A and B atoms occupies the p_z orbital, which is orthogonal the graphene plane. The hybridization of these p_z orbitals provides the formation of the π-bands in graphene. Then, the electron properties of graphene can be described within a single p_z orbital tight-binding (TB) model [13]. Using the TB approximation, one can show that the effective single-electron Hermitian Hamiltonian is reduced to the two-level pseudo-spin one-half system. The projections of pseudo-spin are associated with two sub-lattices.

In graphene, the dispersion relation, $E(\mathbf{k})$ (where $\mathbf{k}$ is the wave vector), has some specific properties: the Fermi level corresponds to $E(\mathbf{k}) = 0$, and the valence and conduction bands touch each other in the first Brillouin zone at six points. Each of these points provides a "conical intersection" known also as the "diabolical point" (DP) [14,15]. (See Fig. 2.)

According to [13], in the $\mathbf{k}$-representation the dynamics of the electron, in the vicinity of the DP, can be described by the following Hamiltonian:

$$\mathcal{H}_0 = \begin{pmatrix} 0 & \hbar v_F (q_x - i q_y) \\ \hbar v_F (q_x + i q_y) & 0 \end{pmatrix}, \tag{1}$$

where $v_F = 3|t|a/2\hbar$ is the Fermi velocity. Here $t \approx -2.8$ eV, is the hopping integral for nearest neighbor atoms, A and B, with coordinates, $\mathbf{R}_A$ and $\mathbf{R}_B$. We use the following notations: the state, $|1\rangle = \binom{1}{0}$, corresponds to the population of the sub-lattice A, and the state, $|2\rangle = \binom{0}{1}$, corresponds to the population of the sub-lattice B. The Hamiltonian, $\mathcal{H}_0$, has the eigenvalues, $E_\pm = \pm\hbar v_F |\mathbf{q}|$. The corresponding eigenstates are: $|+\rangle = (1/\sqrt{2})(e^{-i\varphi/2}|1\rangle + e^{i\varphi/2}|2\rangle)$ (conduction band), and $|-\rangle = (1/\sqrt{2})(e^{-i\varphi/2}|1\rangle - e^{i\varphi/2}|2\rangle)$ (valence band), where $\varphi = \arg(q_x + i q_y)$.

Suppose that each discrete state (related to the sub-lattice, A and B) is coupled to a continuum – continuous energy band, which could originate due to impurities or other mechanisms. Suppose, that we allow an electron to tunnel to the continuum from the sub-lattices, A and B, with the ET rates, Γ_1 and Γ_2, correspondingly. Then, the quantum dynamics of the ET can be described by the following effective non-Hermitian Hamiltonian, $\tilde{\mathcal{H}} = \mathcal{H} - i\mathcal{W}$, where $\mathcal{H}$ is the dressed Hamiltonian, $\mathcal{H}_0$, and

$$\mathcal{W} = \frac{1}{2}\begin{pmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{pmatrix}. \tag{2}$$

We find,

$$\tilde{\mathcal{H}} = \frac{\lambda_0}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \varepsilon - i\Gamma & V^* \\ V & -\varepsilon + i\Gamma \end{pmatrix}, \tag{3}$$

where $\lambda_0 = \varepsilon_0 - i\Gamma_0$, $\varepsilon_0 = \varepsilon_1 + \varepsilon_2$, $\Gamma_0 = (\Gamma_1 + \Gamma_2)/2$, $V = 2\hbar v_F (q_x + i q_y)$, $\varepsilon = \varepsilon_1 - \varepsilon_2$, and $\Gamma = (\Gamma_1 - \Gamma_2)/2$. Here, ε_n, is the renormalized energy of the state, $|n\rangle$, which usually occurs in the non-Hermitian Hamiltonian approach [2–6].

Note, that for pure (undoped) graphene, the "effective electron masses", $\varepsilon_{1,2} = 0$, for both sub-lattices. This allows us to study the ST in the vicinity of the EP. However, due to the finite bandwidths, associated with the sinks, not only the ET rates, $\Gamma_{1,2}$, appear, but also the small "effective electron masses", $\varepsilon_{1,2}$, may occur. The condition, $\varepsilon_{1,2} \neq 0$, takes place also in doped graphene and in semiconductors. Both cases, $\varepsilon_{1,2} = 0$ and $\varepsilon_{1,2} \neq 0$, are analyzed below in relation to the ST.

Below, in numerical simulations, we choose $\hbar = 1$. All energy-dimensional parameters are measured in $\text{ps}^{-1} \approx 0.66$ meV. Time is measured in ps. For example, if we choose the experimentally assessable parameters [13], $|\mathbf{q}| = 10^4$ cm^{-1} and $v_F = 10^8$ cm/s, we have: $|V| = 2$ ps$^{-1} \approx 1.32$ meV. The ET rates are varied in the region: $0 \leq \Gamma_{1,2} \leq 10$ ps^{-1}.

3. Superradiance transition

The bi-orthogonal eigenstates of the effective non-Hermitian Hamiltonian, $\tilde{\mathcal{H}}$, provide a convenient basis in which the eigenvalue problem can be formulated and resolved. The solution of the eigenvalue problem, $\tilde{\mathcal{H}}|u\rangle = \tilde{E}|u\rangle$ and $\langle\tilde{u}|\tilde{\mathcal{H}} = \tilde{E}\langle\tilde{u}|$ (where $|u\rangle$ and $\langle\tilde{u}|$ are the right and left eigenvectors respectively), is given by

$$\tilde{E}_{1,2} = \lambda_0/2 \pm \Omega/2, \tag{4}$$