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On the Landau system in noncommutative phase-space

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ABSTRACT

We consider the Landau system in a canonically noncommutative phase-space. A set of generalized transformations containing scaling parameters is derived which maps the NC problem to an equivalent commutative problem. The energy spectrum admits NC corrections which are computed using the explicit NC variables as well as the commutative-equivalent variables. Their exact matching solidifies the evidence of the equivalence of the two approaches. We also obtain the magnetic length and level degeneracy, which admit NC corrections. We further study the Aharonov–Bohm effect where the phase-shift is found to alter due to noncommutativity and also depends on the scaling parameters.

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1. Introduction

In 1930, Landau analyzed the quantum dynamics of a charged particle moving in a background homogeneous magnetic field (to be referred to as the Landau system hereafter) to show that it poses quantized energy levels [1]. These quantized energy levels, dubbed the Landau levels (LLs), arise in a plethora of important physical scenarios – the integral and fractional quantum Hall effect [2,3], Aharonov–Bohm effect [4], different two-dimensional surfaces [5,6] like graphene [7,8], anyons excitations in a rotating Bose–Einstein condensate [9,10], etc., are to name but a few. Apart from these widespread occurrence, on a more formal note, the Landau problem is perhaps the prototypical example of space quantization where one arrives at a coordinate space following a noncommutative algebra, in rudimentary quantum mechanics.

To briefly review this intriguing behaviour let us consider a charged particle of mass m moving in the plane $\vec{x} = (x_1, x_2)$ in the presence of a constant, perpendicular magnetic field B. The Lagrangian will be

$$L = \frac{m}{2}\dot{\vec{x}}^2 - e\dot{\vec{x}} \cdot \vec{A} \tag{1}$$

with the vector potential in the symmetric gauge given by[2]

$$A_i = -\frac{B}{2}\epsilon_{ij} x_j \tag{2}$$

The Hamiltonian can be written in terms of the gauge invariant observable mechanical momentum $\vec{\pi} = m\dot{\vec{x}} = \vec{p} + e\vec{A}$ as

$$H = \frac{1}{2m}\vec{\pi}^{\,2} \tag{3}$$

Note that $\vec{p}$ is the canonical momentum that may vary with gauge choice. Upon quantization by imposing the usual canonical commutation relations it follows that the operators corresponding to the physical momentum have the non-vanishing quantum commutators $\left[\pi_i^{op}, \pi_j^{op}\right] = i\hbar eB\epsilon_{ij}$, showing that the physical momenta, in presence of a background magnetic field $\vec{B}$, belong to a noncommutative (NC) momentum space. Expressing them in terms of the harmonic oscillator creation and annihilation operators, the energy eigenvalues of the Hamiltonian are the LLs

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c \tag{4}$$

with $\omega_c = (\frac{eB}{m})$, the cyclotron frequency.[3]

In the limit $m \to 0$ with fixed B or equivalently $B >> m$ the mass gap between the Landau levels $\Delta\omega_c$ grows and consequently we get the projection of the whole spectrum onto the lowest LL. In this limit (1) becomes a first order Lagrangian $L_0 = -\frac{B}{2}\dot{x}_i \epsilon_{ij} x_j$ which is already expressed in phase-space with the spatial coordinates x_1, x_2 being the canonically conjugate variables so that $\left[x_i^{op}, x_j^{op}\right] = i\frac{\hbar}{2B}\epsilon_{ij}$. Thus we can conclude that noncommuting coordinates arise in electronic systems constrained to lie in the lowest Landau level.

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[2] We follow the convention $\epsilon_{ij} = -\epsilon_{ji}$, with $\epsilon_{12} = 1$.

[3] Velocity of light is taken $c = 1$ throughout the paper.

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Remarkably, a direct analogy to this simple example arises in string theory with D-branes in background "magnetic fields" [11]. D-brane worldvolume can be shown to become a noncommutative (NC) space and a low-energy effective field theory can be arrived at in the point particle limit, where the string length goes to zero. This is known as a noncommutative field theory (NCFT) [12–20] where the coordinate algebra induces a space–time uncertainty relation and the notion of a spacetime point is replaced by a Planck cell of dimension given by the Planck area $|\theta_{ij}|$. Similar NC spatial geometry is also known to arise in various theories of quantum gravity [21–24].

The low energy limit of this NCFT gives us the noncommutative quantum mechanics (NCQM) [25–44] where, we speculate that some relic of the Planck scale effect may be traced [20,45–48]. It would be indeed intriguing to see if such traces can be found in the Landau system itself which has such deep an analogy with the noncommutativity of space as we have discussed above. This problem was addressed by many authors in the literature [49–59] from different perspectives. In recent years, there have also been speculations of a more elaborate NC phase-space structure [60–62], so we carry out our entire analysis in NC phase-space for completeness and generality. Our results can be readily cast into the special case of configuration-space (spatial) noncommutativity only, by equating the momentum NC parameter to zero.

The primary aim of this paper is to study the Landau problem defined over the four-dimensional NC phase-space where operators corresponding to the canonical pairs, denoted by $(\hat{x}_i\,,\,\hat{p}_i)$ follow NC algebra:

$$\left[\hat{x}_i, \hat{x}_j\right] = i\theta_{ij} = i\theta\epsilon_{ij}\;; \quad \left[\hat{p}_i, \hat{p}_j\right] = i\bar{\theta}_{ij} = i\bar{\theta}\epsilon_{ij}\;;$$
$$\left[\hat{x}_i, \hat{p}_j\right] = i\tilde{\hbar}\delta_{ij}. \tag{5}$$

Here θ and $\bar{\theta}$ denotes the spatial and momentum noncommutative parameters and $\tilde{\hbar} = \hbar(1 + \frac{\theta\bar{\theta}}{4\hbar^2})$ is the effective Planck's constant. The usual approach in the literature to deal with such problems is to form an equivalent commutative description of the NC theory by employing some transformation which relate the NC phase-space variables (and the related operators) to ordinary commutative variables (operators) x_i and p_i satisfying the usual operator Heisenberg algebra

$$\left[x^{op}{}_i, p^{op}{}_j\right] = i\hbar\delta_{ij}\;; \quad \left[x^{op}{}_i\,, x^{op}{}_j\right] = 0 = \left[p^{op}{}_i, p^{op}{}_j\right]. \tag{6}$$

In this paper we first carry out our investigation of the Landau system using NC variables explicitly. Specifically, we check whether the magnetic length l_B of this system and the degeneracy of the Landau levels [63,64] acquire corrections from the NC phase-space structure. Surprisingly, these aspects of the Landau system, though very important in context of various observable effects in experimental condensed matter (e.g., the Hall effect) has not been emphasized much in the contemporary NC literature [49–52,54,56,58, 59]. We also compute the spectrum for the system, i.e., the phase-space NCLL. To verify the consistency of our results, we also work out this NC phase-space spectrum taking the usual approach, i.e., by quantizing the commutative-equivalent Hamiltonian obtained using a set of generalized transformations (which we shall derive in this paper) and confront it with the former. Reassuringly, these two NC spectra match exactly, establishing that the present description of the Landau system is unambiguous. Note that unlike the non-linear maps used in [65,56,66], the change of variables used in this paper to obtain the commutative equivalent Hamiltonian are exact maps. The NC phase-space algebra (5) also differs from the one used in [65,56] where similar energy-spectra for the commutative-equivalent theory have been produced.

However, before delving into the analysis of the NC Landau problem, we first study the consequence of phase-space noncommutativity in another important phenomena concerning the Landau system, namely, the Aharonov–Bohm (AB) effect. The significance of the AB effect lies in the fact that it elevates our notion of the electromagnetic potential from being a convenient mathematical concept in Electrodynamics to a physical quantity in quantum mechanics. AB effect arises when one considers a beam of electrons split into two parts, moving in the vicinity of a solenoid placed perpendicular to the plane of the beam. The recombination of these two beams of electrons results in a phase-shift in the interference pattern which depends on the magnetic flux enclosed by the two alternative beam paths. This phase-shift is observed even though the electron-beams move through regions in space devoid of any magnetic field, and only having non-vanishing vector potential, thus establishing the physicality of the latter. Since the electromagnetic vector potential is fixed by the gauge choice for a given background magnetic field in a way (see equation (2)) that will be essentially altered by noncommutativity, it is imperative to check if the NC framework alters the observed phase-shift non-trivially. Further as we have chosen to work with the symmetric gauge in analogy with the commutative scenario in this paper, we employ the usual approach of mapping the NC Hamiltonian of the theory to an equivalent commutative Hamiltonian (with NC corrections) to study the AB effect.

This article is organized as follows. In the next section we derive a mapping between the NC and commutative sets of variables. In Section 3, we present the study of the Aharonov–Bohm effect in NC phase-space, specifically computing the AB phase. Along the way, we describe the framework of obtaining the commutative-equivalent scenario for a theory defined over the NC phase-space. Section 4, contains the analysis of the Landau system in NC phase-space. We conclude in Section 5.

2. Generalized mapping between noncommutative and commutative variables

In this section, we derive a generalized mapping between the NC and commutative sets of variables [67]. We relate the two sets of variables by the following equations

$$\hat{x}_i = a_{ij}x_j + b_{ij}p_j \tag{7}$$

$$\hat{p}_i = c_{ij}x_j + d_{ij}p_j \tag{8}$$

where a, b, c and d are 2×2 transformation matrices. To determine the conditions that the transformation matrices should satisfy, we use the NC algebra (5) and the commutative algebra (6), which yields

$$ad^T - bc^T = \frac{\tilde{\hbar}}{\hbar} \tag{9}$$

$$ab^T - ba^T = \frac{\theta}{\hbar} \tag{10}$$

$$cd^T - dc^T = \frac{\bar{\theta}}{\hbar} \tag{11}$$

where θ and $\bar{\theta}$ are 2×2 antisymmetric matrices. To proceed further, we assume $a_{ij} = \alpha\delta_{ij}$, $d_{ij} = \beta\delta_{ij}$, where α and β are two scaling constants. With these assumptions, eqs. (10) and (11) give the solutions for the matrices b and c as

$$b_{ij} = -\frac{1}{2\alpha\hbar}\theta_{ij} \tag{12}$$

$$c_{ij} = \frac{1}{2\beta\hbar}\bar{\theta}_{ij}. \tag{13}$$

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