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Different types of nonlinear localized and periodic waves in an erbium-doped fiber system

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ABSTRACT

We study nonlinear waves on a plane-wave background in an erbium-doped fiber system, which is governed by the coupled nonlinear Schrödinger and the Maxwell–Bloch equations. We find that prolific different types of nonlinear localized and periodic waves do exist in the system, including multi-peak soliton, periodic wave, antidark soliton, and W-shaped soliton (as well as the known bright soliton, breather, and rogue wave). In particular, the dynamics of these waves can be extracted from a unified exact solution, and the corresponding existence conditions are presented explicitly. Our results demonstrate the structural diversity of the nonlinear waves in this system.

1. Introduction

Nonlinear waves propagating in an erbium-doped fiber have attracted special attention, since the resonant absorption of the erbium-doped two-level system is a good solution to the fiber-optic signal attenuation problem [1]. In general, the dynamics of nonlinear waves in the erbium-doped fibers is described by the coupled nonlinear Schrödinger and the Maxwell–Bloch (NLS-MB) models [1–3]. Like the standard NLS model, the standard bright (i.e., zero background) solitons [2,3], and the localized waves on a plane-wave background, such as rogue waves [4,5] and breathers [5,6] in the NLS-MB system have been demonstrated.

However, in contrast to the scalar NLS system, the coupled NLS-MB system possesses some additional system parameters and allows for interaction between different components, which potentially yields rich and significant localized-wave dynamics. Indeed, recent studies indicate that rich nonlinear localized structures do exist in the coupled NLS systems [7–15], such as rogue waves with different structures [7–9], coexistence of different types of localized nonlinear waves [10–13], and so on [14,15]. On the other hand, due to the breaking of the Galilean transformation for the plane-wave background fields, the background frequency plays a key role in the types of nonlinear waves and cannot be ignored in the coupled NLS-MB system. In fact, it is demonstrated recently that the different values of the background frequency in the higher-order NLS model (where the Galilean transformation is broken) can induce different types of localized waves [16,17]. Motivated by these results, we shall study prolific types of nonlinear waves in the NLS-MB system.

In this letter, we present intriguing different types of nonlinear localized and periodic waves in an erbium-doped fiber system, including multi-peak soliton, periodic wave, antidark soliton, and W-shaped soliton (as well as the known bright soliton, rogue wave, and breather). It is found that these waves can be extracted from a unified exact solution under specific parameter conditions. In particular, the multi-peak soliton could be regarded as a single pulse formed by a nonlinear superposition of a soliton and a periodic wave, where each have the same velocity.

2. NLS-MB system and different types of nonlinear waves

We consider a resonant erbium-doped fiber system governed by a coupled system of the NLS-MB equations [1–3]

$$E_z = i(\frac{1}{2}E_{tt} + |E|^2E) + 2P,$$
$$P_t = 2i\omega P + 2E\eta,$$
$$\eta_t = -(EP^* + PE^*), \quad (1)$$

where $E(z,t)$ is the slowly varying envelope field; $P(z,t)$ is the measure of the polarization of the resonant medium, which is defined by $P = v_1 v_2^*$; $\eta(z,t)$ denotes the extent of the population inversion, which is given by $\eta = |v_1|^2 - |v_2|^2$, v_1 and v_2 are the

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wave functions of the two energy levels of the resonant atoms; ω is the carrier frequency, and the index $*$ denotes complex conjugate. In order to study abundant types of nonlinear structures in the NLS-MB model in contrast to the previous results [2–6], we first introduce the following plane-wave background solution with a generalized form

$$E_1 = ae^{i\theta},\ P_1 = ikE_1,\ \eta_1 = \omega k - qk/2, \tag{2}$$

where $\theta = qt + \nu z$, $\nu = a^2 + 2k - q^2/2$, a and q represent the amplitude and frequency of background electric field, respectively, and k is a real parameter which is related to the background amplitude of P component. If the background amplitudes vanish, Eq. (2) reduces to the trivial solution, which can be used to generate standard bright soliton solutions [2,3,18,19]. Here we will pay our attention to different types of nonlinear structures in electric field, i.e., the pulse propagation in the E component. We omit the results in the P, η components, since their types of nonlinear waves are similar to the ones in the E component with the same initial physical parameters. We present the first-order exact nonlinear wave on the plane-wave solution (2) to reveal rich different types of nonlinear waves. The construction method is based on the Darboux transformation technique [20] applied to the Lax pair associated with the NLS-MB model. For the details, one can construct the solution by solving the partial differential equations (Lax pair) starting from the initial seed solution (2). After that the general first-order exact nonlinear wave solution on the plane-wave background in the E component is given

$$E = E_1 \left\{ 1 - \frac{8bm_1[\sin(\gamma+\mu_1) - i\sinh(\beta+i\mu_1)]}{m_3\sin(\gamma+\mu_2) - im_2\sinh(\beta-i\mu_3)} \right\}, \tag{3}$$

where

$$\beta = \zeta(t + V_1 z),\ \gamma = \sigma(t + V_2 z),$$
$$V_1 = \upsilon_1 + b\sigma\upsilon_2/\zeta,\ V_2 = \upsilon_1 - b\zeta\upsilon_2/\sigma,$$
$$\upsilon_1 = k\omega/(b^2+\omega^2) - q/2,\ \upsilon_2 = 1 - k/(b^2+\omega^2),$$
$$m_1 = \sqrt{(i\zeta-\sigma)^2 + (2b+iq)^2},$$
$$m_2 = \sqrt{(\alpha_1+\alpha_2)^2 - 4(2b\zeta+\sigma q)^2},$$
$$m_3 = \sqrt{(\alpha_1-\alpha_2)^2 + 4(2b\sigma-\zeta q)^2},$$
$$\alpha_1 = 4a^2+4b^2+q^2,\ \alpha_2 = \zeta^2+\sigma^2,$$
$$\zeta = \left(\sqrt{\chi^2+16q^2b^2}+\chi\right)^{1/2}/\sqrt{2},$$
$$\sigma = \left(\sqrt{\chi^2+16q^2b^2}-\chi\right)^{1/2}/\sqrt{2},$$
$$\chi = 4b^2-4a^2-q^2,\ \mu_1 = \arctan\left(\frac{2b+iq}{\sigma-i\zeta}\right),$$
$$\mu_2 = \arctan\left(\frac{\alpha_1-\alpha_2}{4b\sigma-2q\zeta}\right),\ \mu_3 = \arctan\left(\frac{\alpha_1+\alpha_2}{2iq\sigma-4ib\zeta}\right). \tag{4}$$

The above expressions depend on the background wave amplitudes a, k, the background wave frequency q, the real parameter $b(\neq 0)$, and the frequency ω. Once the structural parameter ω is fixed, we are left with four independent parameters a, k, q, b.

Remarkably, we find that the unified solution (3) describes abundant different types of nonlinear waves through different choices of the system parameters. For the details, we present a table (Table 1) for the types of nonlinear waves and the corresponding chosen parameter conditions. To our knowledge, some new types of nonlinear waves such as multi-peak soliton, periodic wave, antidark soliton, and W-shaped soliton, are found in the system for the first time.

Table 1
Types of nonlinear waves in NLS-MB system.

Nonlinear wave type	Existence condition
Breather	$b^2+\omega^2 \neq k$, $a \neq b$
Rogue wave	$b^2+\omega^2 \neq k$, $a = b$
Multi-peak soliton	$b^2+\omega^2 = k$, $q \neq 0$
Periodic wave	$b^2+\omega^2 = k$, $q = 0$, $a^2 > b^2$
Antidark soliton	$b^2+\omega^2 = k$, $q = 0$, $a^2 < b^2$
W-shaped soliton	$b^2+\omega^2 = k$, $q = 0$, $a = b$

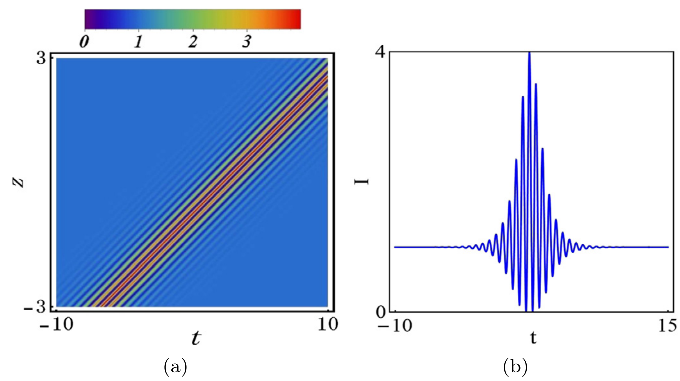

Fig. 1. (a) Intensity distributions ($I = |E|^2$) of multi-peak solitons on a plane-wave background, (b) is the profile of (a) at $z = 0$. The parameters are $a = 1$, $b = 0.5$, $\omega = 2$, and $q = 10$. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

To reveal the properties of these new types of nonlinear structures in the system, let us pay our attention to the explicit function expression of the solution (3). It depends on the hyperbolic functions ($\sinh\beta$) and the trigonometric functions ($\sin\gamma$), where β and γ are real functions of z and t, and V_1, V_2 are the corresponding velocities. In this case, the hyperbolic functions and trigonometric functions describe the localization and the periodicity of the transverse distribution t of the nonlinear waves, respectively. Hence this nonlinear structure could be regarded as a single pulse formed by a nonlinear superposition of a soliton and a periodic wave with velocities V_1, V_2. One should note that the nonlinear wave solution (3) is different from the two-soliton complex solutions [24,25] which mix hyperbolic functions of two different (spatial) arguments. Interestingly, we find that the nonlinear waves described by the unified solution (3) exhibit structural diversity depending on the values of velocity difference, i.e., $V_1 - V_2$.

In the case of nonzero velocity difference, i.e., $V_1 \neq V_2$, which implies $\upsilon_2 \neq 0$ (thus $k \neq b^2+\omega^2$), the expression (3) describes localized waves with breathing behavior on a plane-wave background (i.e., breathers and rogue waves). Here breathers are the localized breathing waves with a periodic profile in a certain direction; rogue waves are rare, short-lived, and localized in both space and time, which are some special cases of breathers. More specifically, if $q = 0$, $\upsilon_2 \neq 0$, we obtain the Akhmediev breathers [21] with $a > |b|$, the Kuznetsov–Ma breathers [22] with $a < |b|$, and the Peregrine rogue waves [23] with $a = b$. We note that the solution (3) includes, as a special case $V_1 \neq V_2$, the breather and rogue wave solutions in the NLS-MB system that was reported in [4–6].

Instead, if $V_1 = V_2$, $q \neq 0$ (thus $\zeta \neq 0$, $\sigma \neq 0$), the pulse described by solution (3) is formed by a nonlinear superposition of a soliton and a periodic wave, where each has the same velocity υ_1. This result is well depicted in Fig. 1. As expected, in this case, the expression (3) describes a new multi-peak soliton-like pulse propagating along z direction. Namely, the feature of this