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Crises in a dissipative bouncing ball model

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ABSTRACT

The dynamics of a bouncing ball model under the influence of dissipation is investigated by using a two-dimensional nonlinear mapping. When high dissipation is considered, the dynamics evolves to different attractors. The evolution of the basins of the attracting fixed points is characterized, as we vary the control parameters. Crises between the attractors and their boundaries are observed. We found that the multiple attractors are intertwined, and when the boundary crisis between their stable and unstable manifolds occurs, it creates a successive mechanism of destruction for all attractors originated by the sinks. Also, a physical impact crisis is described, an important mechanism in the reduction of the number of attractors.

1. Introduction

Modeling of dynamical systems is one of the most embracing areas of interest among physicists and mathematicians in general [1]. Very popular among these models are low-dimensional systems [2,3], whose complex dynamics leading to a rich variety of nonlinear phenomena [3–6], including bifurcations in non-smooth dynamical systems [7].

Here we study the problem of a bouncing ball model, where a free particle is suffering collisions with a vibrating wall under the presence of a constant gravitational field. Holmes [8,9] and Pustylnikov [10,11] were among the first to study the bouncing ball dynamics. This model has been used in many physical and engineering applications. For instance, it describes a similar acceleration phenomenon that cosmic rays experience to acquire high energies, known as Fermi acceleration [12] (considered as the first attempt of prototype for the bouncing ball dynamics); the dynamic stability in human performance, where a human tries to stabilize a ball on a vibrating tennis racket [13]; and the subharmonic vibration waves in a nanometer-sized mechanical contact system [14]. One can also find studies in granular materials [15–18], experimental devices concerning normal coefficient of restitution [19,20], mechanical vibrations [21–23], anomalous transport and diffusion [24,25], thermodynamics [26], chaos control [27–29], besides the well known connection with the standard mapping [2], which leads to other several applications.

Although the bouncing ball problem has been studied for many years [8–11,30,31], concerning different aspects and applications, the implications of the nonlinear perturbation requires an extensive and complex analysis where some chaotic properties are not yet fully understood. In this paper we consider a high dissipative bouncing ball model where a coefficient of restitution plays the role of dissipation, and the perturbation parameter is physically interpreted as a ratio between the moving plate acceleration and the gravitational field. For some combinations of parameters, plenty of attractors can coexist [32–34]. We found that these attractors in the phase space are intertwined, and varying the value of the control parameter of perturbation, we characterize a boundary crisis [6,35–37] between the stable and unstable manifold of the same saddle point. Such a crisis leads to successive destruction of these intertwined attractors and is a mechanism that allows the lowest energy attractor, which is related to the vibrating wall, to continue to exist, giving it the status of a robust attractor. In addition, we describe a physical impact crisis, between the real vibrating plate and the border of an attractor. This crisis, as yet unclassified, reduces the number of attractors dramatically at a single parameter value.

The organization of the paper is given as follows. In Section 2 we describe the dynamical system under study and its chaotic properties. Section 3.1 is devoted to the numerical analysis of the average velocity, in Section 3.2 we study the basin of attraction of the fixed points and set up the impact physical crisis, and in Section 3.3 we discuss the relation between the manifolds boundary crisis and the attractors; finally in Section 4 we draw some final remarks and conclusions.

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2. The model, the mapping and chaotic properties

In this section we describe the model under study, the bouncing model, which consists of a particle, under the influence of a constant gravitational field, that suffers inelastic collisions with a heavy oscillating wall. Dissipation is introduced via a restitution coefficient $\gamma \in [0,1]$, where $\gamma = 1$ recovers the conservative case, where Fermi Acceleration (FA) is inherent [25,38]. The introduction of dissipation can be considered as a suppression mechanism for this unlimited energy growth [39,40]. The system is oriented along the vertical axis, where the upward direction is said to be positive, the wall equilibrium position is set at $y = 0$, and the dynamics is basically described by a non-linear mapping for the variables velocity of the particle v and time t immediately after a nth collision of the particle with the vibrating wall.

There are two distinct versions of the dynamics description: (i) complete one, which consists in considering the complete movement of the time-dependent wall, and (ii) simplified, where the wall is assumed to be fixed, but exchanges momentum and energy with the particle upon collision. Both approaches produce a very similar dynamic considering conservative [25] and dissipative cases [26,39–41]. In the complete version, the vibrating wall obeys the equation $y_w(t_n) = \varepsilon \cos wt_n$, where ε and w are respectively, the amplitude and the frequency of oscillation of the vibrating wall. In the simplified version, the vibrating wall is said to be fixed at $y = 0$, but when the particle collides with it, they exchange momentum and energy as if the wall were vibrating. Thus, the simplified approach keeps the nonlinearity of the model and significantly speeds up the numerical simulations, as well allows easier analytical calculations. In this paper and from this point beyond, we only deal with the complete version of the mapping.

Considering the flight time, which is the time that the particle spends to go up, stop with zero velocity, starts falling and collides again with the vibrating wall, we define some dimensionless and more convenient variables as: $V_n = v_n w/g$, $\epsilon = \varepsilon w^2/g$, where V_n is the "new dimensionless velocity", g is the gravitational field and ϵ can be understood as a ratio between accelerations of the vibrating wall and the gravitational field. For instance, one can set some real values for the dimensional variables, as $g = 10$ m/s^2, $\varepsilon = 0.001$ m, $w = 2\pi f$, where $f = 100$ Hz, and obtain the dimensionless variable $\epsilon \approx 0.1591$. Some real devices concerning impact experiments with granular material can be found in Refs. [19,20]. Also, measuring the time in terms of the number of oscillations of the vibrating wall $\phi_n = wt_n$, we obtain the mapping

$$T: \begin{cases} V_{n+1} = -\gamma(V_n^* - \phi_c) - (1+\gamma)\epsilon \sin(\phi_{n+1}), \\ \phi_{n+1} = [\phi_n + \Delta T_n] \mod (2\pi), \end{cases} \quad (1)$$

where the expressions for V_n^* and ΔT_n depend on the kind of the considered collision. For the case of multiple collisions inside the collision zone $[-\epsilon, +\epsilon]$, the expressions are $V_n^* = V_n$ and $\Delta T_n = \phi_c$ where ϕ_c is obtained from the condition that matches the same position for the particle and the vibrating wall, expressed as

$$G(\phi_c) = \epsilon \cos(\phi_n + \phi_c) - \epsilon \cos(\phi_n) - V_n \phi_c + \frac{1}{2}\phi_c^2, \quad (2)$$

where this transcendental equation must be solved numerically for $G(\phi_c) = 0$, with $\phi_c \in (0, 2\pi]$.

If the particle leaves the collision zone case after a collision, goes up, reach null velocity, and falls for an another collision, we have indirect collisions and the expressions are $V_n^* = -\sqrt{V_n^2 + 2\epsilon(\cos(\phi_n) - 1)}$ and $\Delta T_n = \phi_u + \phi_d + \phi_c$ with $\phi_u = V_n$ denoting the time spent by the particle in the upward direction up to reaching the null velocity, $\phi_d = \sqrt{V_n^2 + 2\epsilon(\cos(\phi_n) - 1)}$ corresponds to the time that the particle spends from the place where it had zero velocity up to the entrance of the collision zone at ϵ.

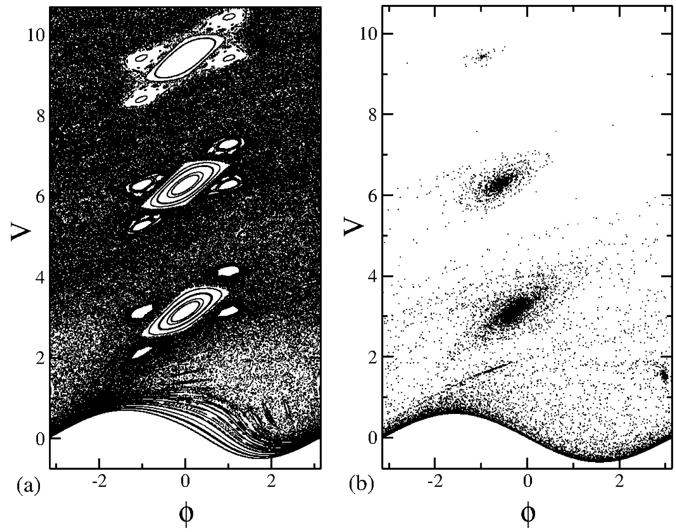

Fig. 1. Comparison between phase space for conservative and dissipative dynamics. In (a) $\epsilon = 0.6$ and $\gamma = 1.0$, and in (b) $\epsilon = 0.6$ and $\gamma = 0.9$. In (b) the thick black regions are the sinks and the bottom attractor. Also, all the spread dots are the transient.

Finally the term ϕ_c has to be obtained numerically from the equation

$$F(\phi_c) = \epsilon \cos(\phi_n + \phi_u + \phi_d + \phi_c) - \epsilon - V_n^* \phi_c + \frac{1}{2}\phi_c^2, \quad (3)$$

where $F(\phi_c)$ represents a transcendental equation that must be solved numerically in order to find the exact "time" of collision, as $F(\phi_c) = 0$, with $\phi_c \in [0, 2\pi]$.

The obtainment of the numerical root ϕ_c is done considering at first $G(\phi_c) = 0$. If we did not find any root for $G(\phi_c)$, we start to evaluate $F(\phi_c) = 0$. The root seeking process is made by solving the transcendental equations via bisection method, with a precision of 10^{-14}.

Taking the determinant of the Jacobian matrix of both kinds of collisions (see Ref. [40] for details), and after a straightforward algebra, it is easy to show that the mapping (1) shrinks the phase space measure since the determinant of the Jacobian matrix is given by

$$\mathrm{Det\,J} = \gamma^2 \left[\frac{V_n + \epsilon \sin(\phi_n)}{V_{n+1} + \epsilon \sin(\phi_{n+1})} \right]. \quad (4)$$

Here, if $\gamma = 1$ we recover the non-dissipative version of the mapping, in fact, as velocity and phase are not canonical pairs in the complete version, the determinant of J is not the unity, but rather it leads to the following measure to be preserved, $d\mu = (V + \epsilon \sin\phi) dV d\phi$. Indeed, the extended phase space for the whole version of the model considers four variables namely: (1) y_w denoting the position of the vibrating wall; (2) V_p corresponding to the velocity of the particle; (3) E_p which is the mechanical energy (kinetic+gravitational) of the particle and (4) the time t. The canonical pairs however are: position and velocity (y_w, V_p) and energy and time (E_p, t).

Another useful property for the dynamics evolution, as function of the control parameters, is the analysis of the fixed points and their stability. For the bouncing ball model the period-1 fixed points can be obtained by doing $V_{n+1} = V_n = V^*$ and $\phi_{n+1} = \phi_n = \phi^* + 2m\pi$ in Eq. (1). For both kinds of collisions, successive and indirect, the fixed points are

$$V^* = m\pi \ ; m = 1, 2, \ldots, \quad \phi^* = \arcsin\left(\frac{V^*(\gamma - 1)}{(1+\gamma)\epsilon}\right). \quad (5)$$