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Bi- and uni-photon entanglement in two-way cascaded fiber-coupled atom–cavity systems

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ABSTRACT

We theoretically investigate the two-photon entanglement in fiber-coupled, two-way cascaded atom–cavity systems. In particular, we demonstrate that, it is possible to generate two-photon entanglement in both weak coupling (atom–cavity coupling rate $|g|$ smaller than the cavity leakage rate κ) and strong coupling regimes ($\kappa < |g|$) in this system, when both atoms start off in an excited state. By employing the quantum trajectory method, we characterize the two-photon entanglement in terms of von-Neumann entropy and show that the amount of entanglement exceed considerably (almost double) when $\kappa > |g|$. We also quantified the amount of entanglement when instead of two excitations there is a single excitation in the system in the beginning.

1. Introduction

Entanglement generation and its preservation against unwanted environmental noise is one of the key challenges for reliable long distance quantum communication and quantum information protocols [1,2]. In such protocols an entanglement has to be established among different nodes of a quantum network which can then be used as a resource of information [3]. Photons are one promising candidate which can be utilized as flying qubits which can mediate entanglement among distant quantum systems. With the development of optical fiber technology, it is now possible to even guide such single and multi-photonic states of light from one point to the other, with well controlled spectral and temporal profiles.

Along with the fiber optics technology, last decade or so has also witnessed a huge amount of progress in the manufacturing of optical cavities (in many geometries) with an incredibly high Q-values [4]. Moreover, through the simultaneous progress in the ion-trapping and solid state quantum dots techniques, it is also now possible to couple these high quality factor optical cavities strongly with few or many atoms to perform various cavity-QED experiments. A more recent area in this context is of coupled atom–cavity arrays (CCA) in which many atom–cavity systems are networked together through optical fiber(s) in various geometries and dimensions [5]. CCA has a wide range of applications such as: creating novel quantum states of light and matter [6], implementation of various spin chain models [7], observation of many-body effects with optical systems [8] and storage/delay of classical and quantum light [9,10].

In this article, we study in what ways we can probe the coherence-preserving or coherence-destroying properties of the coupled cavity arrays. In our recent study we have investigated the influence of non-linearities produced by atom–cavity coupling strengths on the presence of quantum interference effects (Hong–Ou–Mandel-like interference at two photon level) in a two atom–cavity coupled system [11]. Such a study is crucial when single photons are stored in a CCA and we are interested to notice changes in spectral and temporal properties of retrieved photons.

In present work, we study entanglement in CCA, because it is known that entanglement is most sensitive to loss of coherence. In the discussion of entanglement, we notice that our system (as shown in Fig. 1) consists of four cavity modes and two atoms distributed evenly over two locations, bipartite entanglement of different types can occur: between the cavity modes, between the atoms, and hybrid entanglement between atom and cavity modes. For example, mode entanglement may be of the form $(|0\rangle_{a_1}|1\rangle_{a_2}|1\rangle_{a_3}|0\rangle_{a_4} - |1\rangle_{a_1}|0\rangle_{a_2}|0\rangle_{a_3}|1\rangle_{a_4})/\sqrt{2}$, which can be interpreted as entanglement between the two photons (one on each side; the subscripts here indicate the four counter propagating modes in the two cavities as indicated in Fig. 1), or, alternatively, of the form $(|0\rangle_L|2\rangle_R - |2\rangle_L|0\rangle_R)/\sqrt{2}$, which occurs in the HOM effect [12], and which cannot be interpreted as entanglement between the two photons (instead, it is the modes that are entangled [13]). In this state subscripts are showing the left (L) and right (R)

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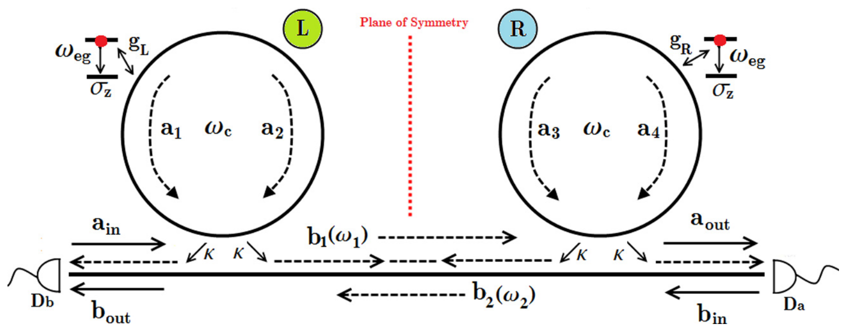

Fig. 1. Two spatially separated atom–cavity systems, and two single-photon detectors (same setup used in our other recent work [11]). The detectors count photons in two output modes, described by annihilation operators $\hat{a}_{out}$ and $\hat{b}_{out}$. As a result of initially excited atoms, at a later time there can be two photons in the system which can transfer from one atom–cavity system to the other through bi-directional cascaded coupling of the fiber with the modes of the cavities. Note that for the sake of simplicity we have considered a mirror symmetric case when both atom–cavity systems are assumed to be identical (for further details about the system see text).

atom–cavity systems as shown in Fig. 1. The HOM type of two photons state can also exist between left and right atom–cavity system as well. Here we would also like to point out that throughout this paper, by single and two photon entanglement we mean that photons being the mediator of entanglement, entangle atoms or cavities or atom–cavity system.

We find that using two fiber coupled atom–cavity systems, in which both of the atoms are initially in an excited state, it is possible to generate two-photon mode entanglement. In the language of quantum jump approach the time till which both photons remain inside the system, the system evolves in a pure no-jump state and hence Von Neumann entropy [14] is a suitable measure to quantify the amount of entanglement in this scenario. We find the weak coupling regime is a more favorable choice for producing larger two-photon entanglement which eventually dies out due to the photon leakage through optical cavities. We also discussed the case when instead two photons there is single excitation in the system to begin with. And this single photon, as an entanglement mediator can entangle atoms, cavities and atom–cavity systems for considerable amount of time. Here we would like to emphasize that there are a number of other studies performed in recent past in which single photon entanglement among atoms or cavities in a fiber coupled atom–cavity system is investigated [15–26]. The novelty of our work relies on the fact that we have addressed two-photon entanglement in a two-way cascaded resonator–atom setups within the framework of quantum trajectory method. And even in the case of single photon entanglement, we have also studied how to entangle two hybrid atom–cavity systems instead of just two atoms or two cavity modes, which to our knowledge has not been reported in the present context so far.

The paper is organized as follows. In Section 2 we describe the model of the system under consideration. Dissipative dynamics of the system as described in terms of quantum trajectory method is discussed in Section 3. In Section 4 we analyze various types of entanglement occurring in our system and investigate the influence of different parameters on the entanglement dynamics. Finally in Section 5 we present the conclusions of this work.

2. Model and Hamiltonian

The setup consisting of two atom–cavity systems is shown in Fig. 1. Atoms are two-level systems with a ground (excited) state $|g\rangle$ ($|e\rangle$) and an atomic transition frequency ω_{eg}. Both atoms are assumed to be initially excited and the spontaneous emission events are completely ignored. Depending on the directions in which photons are emitted inside cavities, any one of the two counter propagating modes in each cavity can be excited. In the left cavity these modes have a single resonant frequency ω_c and these are described by the annihilation operators $\hat{a}_1$ and $\hat{a}_2$. For the right side cavity the resonant frequency is the same (as for left cavity) but modes are represented by the operators $\hat{a}_3$ and $\hat{a}_4$.

Atom–cavity coupling strength in left and right systems is given by the rates g_L and g_R respectively and cavities are also coupled with a dispersion-less optical fiber. The fiber is assumed to have two continua of modes. One continuum, expressed through the continuum operator $\hat{b}_1(\omega_1)$ ($\hat{b}_2(\omega_2)$) couples the fiber modes to the right (left) going photons. Photon leakage from the cavities occur at a rate κ which not only causes photons to move back and forth between these two atom–cavity systems but also guides the photons towards the output detectors (D_a and D_b). Note that the assumption of all cavity decay rates to be the same is not just made for simplicity here, rather the presence of time-reversal symmetry in our system (which can be seen in Fig. 1), forces us to make this choice. The combined system, fiber and system-fiber interaction Hamiltonian under rotating wave and dipole approximations can be expressed as:

$$\begin{aligned}\hat{H}/\hbar = &-\omega_{eg}\hat{\sigma}_-^{(L)}\hat{\sigma}_+^{(L)} - \omega_{eg}\hat{\sigma}_-^{(R)}\hat{\sigma}_+^{(R)}\\ &+\omega_c(\hat{a}_1^\dagger\hat{a}_1 + \hat{a}_2^\dagger\hat{a}_2 + \hat{a}_3^\dagger\hat{a}_3 + \hat{a}_4^\dagger\hat{a}_4)\\ &+(g_L\hat{a}_1^\dagger\hat{\sigma}_-^{(L)} + g_L^*\hat{a}_1\hat{\sigma}_+^{(L)}) + (g_L^*\hat{a}_2^\dagger\hat{\sigma}_-^{(L)} + g_L\hat{a}_2\hat{\sigma}_+^{(L)})\\ &+(g_R\hat{a}_3^\dagger\hat{\sigma}_-^{(R)} + g_R^*\hat{a}_3\hat{\sigma}_+^{(R)})\\ &+(g_R^*\hat{a}_4^\dagger\hat{\sigma}_-^{(R)} + g_R\hat{a}_4\hat{\sigma}_+^{(R)}) + \int_{-\infty}^{+\infty}\omega_1\hat{b}_1^\dagger(\omega_1)\hat{b}_1(\omega_1)d\omega_1\\ &+\int_{-\infty}^{+\infty}\omega_2\hat{b}_2^\dagger(\omega_2)\hat{b}_2(\omega_2)d\omega_2\\ &+i\sqrt{\frac{\kappa}{2\pi}}\int_{-\infty}^{+\infty}\Big(\hat{a}_1\hat{b}_1^\dagger(\omega_1) - \hat{a}_1^\dagger\hat{b}_1(\omega_1)\\ &+\hat{a}_3\hat{b}_1^\dagger(\omega_1) - \hat{a}_3^\dagger\hat{b}_1(\omega_1)\Big)d\omega_1\\ &+i\sqrt{\frac{\kappa}{2\pi}}\int_{-\infty}^{+\infty}\Big(\hat{a}_2\hat{b}_2^\dagger(\omega_2) - \hat{a}_2^\dagger\hat{b}_2(\omega_2)\\ &+\hat{a}_4\hat{b}_2^\dagger(\omega_2) - \hat{a}_4^\dagger\hat{b}_2(\omega_2)\Big)d\omega_2. \end{aligned} \tag{1}$$

Here $\hat{\sigma}_+^{(L)}, \hat{\sigma}_+^{(R)}$ are the atomic raising operators for left and right atoms respectively and the non-vanishing commutation relations are: $[\hat{\sigma}_+^{(L)}, \hat{\sigma}_-^{(L)}] = \hat{\sigma}_z^{(L)}$ and a similar relation for right atom and