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Quantum computation with classical light: The Deutsch Algorithm

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ABSTRACT

We present an implementation of the Deutsch Algorithm using linear optical elements and laser light. We encoded two quantum bits in form of superpositions of electromagnetic fields in two degrees of freedom of the beam: its polarisation and orbital angular momentum. Our approach, based on a Sagnac interferometer, offers outstanding stability and demonstrates that optical quantum computation is possible using classical states of light.

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1. Introduction

Quantum computation processes information encoded in quantum systems and promises unprecedented computational power by exploiting features of quantum mechanics. Since its invention three decades ago [1,2], the fascinating concept has been developed into a vibrant research field linking physics and computer science with mathematics. A variety of quantum algorithms [3] and experimental implementation techniques [4] have been explored. However, due to the difficulty of the task to coherently control and individually address a multitude of quantum systems necessary in order to improve on the speed of the best classical computers, the science community is still waiting for the first numerical result calculated by a quantum computer that is unattainable by the current conventional computer technology.

In theory it was proven already, that certain computational problems based on the use of oracles can be solved efficiently by means of quantum algorithms but not using classical ones [5]. Results in connection with period finding and the Hidden Subgroup Problem [6], for example Shor's algorithm to factorize numbers [7,8], indicate that there are also non-oracular algorithms that can only be solved efficiently on a quantum computer.

What makes quantum computation more efficient than classical computation? Representing inputs in terms of basis states of quantum systems allows to process in parallel many inputs in form of superpositions of the basis states. However, the resulting superposition of transformed basis states representing the processed inputs would be destroyed when measured directly. Therefore, the read-out of the processed information requires careful exploitation of interference effects. Moreover, entanglement is an important ingredient of quantum computation for pure states [9] but its presence is not necessary when computing with mixed states [10] as can already be seen from the first proof-of-principle implementation of the Shor Algorithm using nuclear magnetic resonance and pseudo-pure states without entanglement [11].

Also in classical optics the ingredients of quantum computing, as described above, i.e., superpositions, interference and indeed a form of entanglement exist. The similarities between paraxial optics and non-relativistic quantum mechanics [12] can be used to describe optics in terms of the Dirac formalism [13] and solve problems from Fourier optics elegantly using operator algebra methods from quantum mechanics [14–16]. A form of entanglement is present in classical vector beams [17–21], where the polarisation cannot be separated from the spatial dependence of the electric field [22]. This "classical" entanglement has been

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used to determine the class of physically realizable Mueller matrices [23], to distinguish vector beams from scalar beams [22], and it assisted in designing schemes to realize quantum walks with classical (laser) light [24]. The recent discovery of classical entanglement has completed the list of optical ingredients which enable quantum computation.

Therefore, it is interesting to study to which extent quantum computation can be realized by means of classical optics. Here, as a first step, we apply the similarities between classical optics and quantum mechanics to demonstrate an experimental implementation of a simple quantum algorithm, the Deutsch Algorithm, with classical light. The experiment serves as a proof of principle that an oracle-based quantum algorithm can be implemented with classical light analogously to quantum systems and with the same speed-up compared to classical algorithms.

The Deutsch Algorithm can be implemented by means of different quantum systems, including cavity QED [25–27], atomic ensembles [28], quantum dots [29,30] and nuclear spins [31]. Moreover, there are several purely optical schemes which employ the polarisation and spatial modes as input and output registers for the Deutsch Algorithm. For instance, Oliveira et al. [32] used Hermite–Gaussian modes in a Mach–Zehnder interferometer to perform the algorithm, however the natural instability of the setup required additional equipment to avoid noisy results. Another approach [33] is based on a Sagnac configuration with paths qubits at the single-photon level to execute the computation. More recently, Zhang et al. suggested an implementation with a control gate conditioning the Orbital Angular Momentum (OAM) of a single photon on its polarisation state by means of a q-plate [34]. While some of these attempts already employ classical states of light (from a bright [32] and a weak laser source [33]) the authors argue that the computation is carried out by single photons many times in parallel [32] or individually [33]. Here we demonstrate using Laguerre–Gaussian (LG) beams in a stable Sagnac interferometer that classical light can be used directly to perform certain quantum computations without the need to prepare cumbersomely quantum states of light.

2. Concept and theory

There is a vast literature on quantum computation and quantum information treating in great detail the Deutsch problem [4, 35–37]. For instance, Audretsch [35] gives a good introduction. Even so, we will briefly outline the problem and its solution. Assume that a function

$$f : \{0,1\} \rightarrow \{0,1\} \quad (1)$$

can be accessed only via an oracle, i.e., a black box that computes the functional values $f(x)$ given the arguments x. The problem is to determine a global property of the function: whether it is constant ($f(x) = \text{const}$) or balanced (f assumes the values 0 and 1). In order to classically solve this problem, we need to query the black box at least twice. In the quantum realm, the Deutsch Algorithm allows us to determine this property in a single measurement.

f can be one of the following functions

$$\begin{aligned} f_1(0) &= 0, \quad f_1(1) = 0, \\ f_2(0) &= 1, \quad f_2(1) = 1, \\ f_3(0) &= 0, \quad f_3(1) = 1, \\ f_4(0) &= 1, \quad f_4(1) = 0, \end{aligned} \quad (2)$$

where f_1 and f_2 are constant, while f_3 and f_4 are balanced functions.

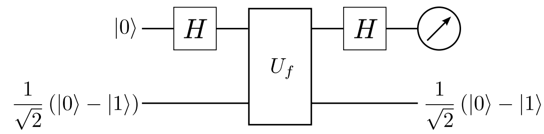

Fig. 1. Quantum circuit for the Deutsch Algorithm. Here H stands for the Hadamard transformation and U_f represents the oracle. The measurement is carried out in the first register.

In order to determine the function via a quantum oracle the implementation of a unitary transformation U_f is mandatory, and its computation is done according to the rules of modular arithmetic

$$|x, y\rangle \xrightarrow{U_f} |x, y \oplus f(x)\rangle, \quad (3)$$

where $\oplus$ represents addition modulo 2 and the state vector $|x, y\rangle \equiv |x\rangle \otimes |y\rangle$ is the tensor product of the state of the first register, $|x\rangle$, with the state of the second register, $|y\rangle$. We observe that the state of the second register is dependent on the first register, in that sense U_f is said to be a controlled operation. Furthermore, the Deutsch Algorithm employs a Hadamard gate before and after the call of the quantum oracle U_f. In particular, the Hadamard gate acts on the computational basis $|0\rangle$, $|1\rangle$ as follows: $|0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and $|1\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

The sequence of gates in the algorithm is described by the quantum circuit shown in Fig. 1. The required input state reads

$$|\Psi_{\text{in}}\rangle = |0\rangle \left[\frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \right]. \quad (4)$$

After the quantum circuit we obtain the state [2,38] (see Appendix A)

$$|\Psi_{\text{out}}\rangle = \left[\left((-1)^{f(0)} + (-1)^{f(1)} \right) |0\rangle + \left((-1)^{f(0)} - (-1)^{f(1)} \right) |1\rangle \right] \times \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle). \quad (5)$$

From this state we can observe that if $f(0) = f(1)$ there is constructive interference on $|0\rangle$, and destructive interference on the $|1\rangle$ component for the first register. The opposite is true for $f(0) \neq f(1)$. Finally, a measurement of the computational basis in the first register reveals the answer; if we obtain a result corresponding to $|0\rangle$ the function is constant, otherwise, if we detect $|1\rangle$, the function is balanced.

To implement the algorithm we need to encode qubits in two registers. For the first register we choose the linear polarisations, horizontal and vertical, as computational basis. In the second register we use the OAM degree of freedom of LG beams to represent the computational basis states. LG beams belong to the class of helical doughnut modes (see Fig. 2) with a phase dependence of $\exp(il\theta)$, where θ is the azimuthal angle, l is known as the topological charge and carries an OAM of $l\hbar$ per photon [39–41]. The sign of l indicates the handedness of the helical (or screw-like) wavefront of the beam. A LG mode (of zero radial order) at the plane $z = 0$ may be written as [42]

$$\langle r, \theta \,|l\rangle \equiv \varphi_l(r,\theta) = (r/w_0)^{|l|} \exp(-r^2/w_0^2) \exp(il\theta), \quad (6)$$

where r is the radial coordinate and w_0 is the beam waist. Note that helical modes with different angular indices l, l' are orthogonal:

$$\langle l' \,|l\rangle = \iint_{\mathbb{R}^2} \varphi^*_{l'}(r,\theta)\varphi_l(r,\theta)\, r\, \mathrm{d}r\, \mathrm{d}\theta = \delta_{l'l}, \quad (7)$$

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