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Precisely controllable bright nonautonomous solitons in Bose–Einstein condensate

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ABSTRACT

We study on bright nonautonomous solitons of Bose–Einstein condensate analytically in a time-dependent harmonic trap with an arbitrary time-dependent linear potential and complex potential. The explicit ways to control dynamics of soliton are presented through observing the evolution of its width, peak, and the motion of its center analytically. Furthermore, we present two-solitons solution in generalized form to observe the collision of solitons conveniently.

1. Introduction

There have been great interest in studying the various properties of ultra cold atoms with the experimental realization of trapped Bose–Einstein condensates (BECs) in alkali atoms [1,2]. The formation and propagation of matter wave solitons are more interesting dynamical feature in Bose–Einstein condensates, such as dark solitons [3–8], bright solitons [9–12], and four-wave mixing [13]. Recently, there are some experimental reports on solitons in Bose–Einstein condensate (BEC) system [14–17], which would inspire the studies of solitons in applications. To make soliton use in practice, we must find some operations which affect the dynamics of soliton in BEC, such as trap potentials and interaction between atoms and so on. Dynamics of solitons of condensate in a harmonic trap and complex potential has been studied in many papers [18]. When the frequency of harmonic trap is changed with time and the interaction of atoms is varied via Feshbach resonance, it is found that solitons can exist under some certain conditions, and they evolve with varying amplitudes and speeds, which can be seen as the nonautonomous solitons [19]. Then, how to manage the evolution of solitons through controlling the related parameters? The question deserves further research for soliton application.

At the mean-field level, the Gross–Pitaevskii equation (GPE) governs the evolution of the macroscopic wave function of Bose–Einstein condensates at absolute zero temperature. Considering the atoms transformed from condensates to thermal could, it is suitable to add a complex potential in GPE to describe its effects on soliton of condensate [20]. Besides this, for atoms in the nK ∽ mK temperature regime, the effect of the Earth's gravitational field is by no means negligible especially in the case of magnetic trapping [21]. To describe the effects of gravitational field or other linear potentials, we can introduce an arbitrary time-dependent linear potential in GPE to study the influence of them conveniently. For the cigar-shaped condensate in harmonic trap, one can make the radial frequency much larger than the axial frequency and strongly confine the radial motion. The axial of cylindrical harmonic trap is along the x direction which can be either confining or expulsive. Then, dynamics of the condensate can be described well by the following generalized $(1+1)$-dimensional GPE given as

$$i\frac{\partial\psi(x,t)}{\partial t}+\frac{\partial^2\psi(x,t)}{\partial x^2}+2R(t)\left|\psi(x,t)\right|^2\psi(x,t)+\left[M(t)x^2+f(t)x\right]\psi(x,t)+i\frac{G(t)}{2}\psi(x,t)=0, \tag{1}$$

where $R(t)$ is nonlinearity management parameter which describes the variation of scattering length and can be controlled well by Feshbach resonance [22], $G(t)$ is appropriate gain $(G(t)<0)$ or loss $(G(t)>0)$ terms which can be phenomenologically incorporated to account for the interaction of atomic cloud or thermal cloud. $M(t)x^2$ means a time-dependent harmonic trap and $f(t)x$ stands for an arbitrary time-dependent linear potential. Note that we are using standard notation, with both the fields and coordi-

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nates dimensionless. The similar equations has been solved exactly by many different methods in [19]. However, as far as we know, different methods bring different soliton solutions, and there still lack systemic studies on the dynamics and kinematics of solitons. To make soliton into application, it is meaningful to realize soliton management theoretically and experimentally.

In this Letter, we present single soliton of BECs in a time-dependent harmonic trap through Darboux transformation method, with an arbitrary time-dependent linear potential and complex potential. Dynamics and kinematics of solitons are studied in detail, through observing its shape and motion analytically. Based on the expressions which describe the evolution of soliton's properties, and the compatibility condition, soliton management could be realized theoretically. It is believed that these results would stimulate some experiments to manage soliton. To observe the collision of solitons, we present two-soliton solution in generalized form.

2. The dynamics and kinematics of single bright soliton

According to the Painlevé analysis [23], $R(t)$ and $G(t)$ are not allowed to be space-dependent. We find that under the condition $R(t) = g\exp[\int G(t) - 4C_2(t)\,dt]$ (g is a real non-zero number), where $C_2(t)$ satisfies $4C_2^2(t) + \frac{dC_2(t)}{dt} = M(t)$, soliton solution of Eq. (1) can be achieved by the Darboux transformation method [24] from the Lax-pair[1]

$$\psi(x,t) = \frac{4b(t)\sigma(x,t)^*}{\sqrt{g}(1+|\sigma(x,t)|^2)}\exp\theta(x,t) \tag{2}$$

where

$$\theta(x,t) = iC_2(t)x^2 + \int\big[-G(t)/2 + 2C_2(t)\big]\,dt,$$

$$\sigma(x,t) = A_c\exp\left[-2b(t)x - i2d(t)x - \int 4ib(t)^2\,dt\right] \times \exp\left[\int 4id(t)^2 + 8b(t)d(t)\,dt\right]$$

and

$$b(t) = \alpha\times\exp\left[\int -4C_2(t)\,dt\right],$$

$$d(t) = \left[\int \frac{f(t)}{2}e^{\int 4C_2(t)\,dt}\,dt + \beta\right] \times\exp\left[\int -4C_2(t)\,dt\right]$$

where A_c, α, β, and g are real numbers which relate with the initial condition of soliton, such as initial coordinate, initial velocity, and initial shape. This is a normal solution which can be used to study the properties of bright solitons in BEC trapped in many kinds of potentials and many other systems. Therefore, we are convinced that the soliton can exist in a time-dependent potential with the interaction strength changes with time, which agrees with Serkin and Kumar [19]. More importantly, in this Letter, we will try to find explicit ways to control the evolution of soliton. From the soliton solution, we can calculate nonautonomous soliton's peak, width and the motion of its wave center by assuming that the maximum value of density correspond to the wave center, and the half-value width is the width of soliton. The evolution of them can be given as following (with $C_2(t)$ is a real function): the evolution of width is

$$W(t) = \frac{\ln(3+2\sqrt{2})}{2\alpha}\exp\left[\int 4C_2(t)\,dt\right], \tag{3}$$

the evolution of its peak is

$$|U|^2_{\max} = \frac{4\alpha^2}{g}\exp\left[\int -4C_2(t) - G(t)\,dt\right], \tag{4}$$

and the motion of its wave center is

$$x_c(t) = \frac{\int 4[\int \frac{f(t)}{2}e^{\int 4C_2(t)\,dt}\,dt + \beta]\exp[\int -8C_2(t)\,dt]\,dt}{\exp[\int -4C_2(t)\,dt]} + \frac{\ln A_c}{2\alpha}\exp\left[\int 4C_2(t)\,dt\right]. \tag{5}$$

From the explicit expressions which describe the main properties of solitons, it is convenient to study the effects of each physical operation on the nonautonomous solitons. When the explicit operations are chosen, the corresponding soliton solution can be given directly. Moreover, the evolution of soliton' shape and motion can be investigated analytically through the above expressions which describe them. This provides many possibilities to control the evolution of soliton exactly. The following discussion can be made from the above expressions.

(1) From Eq. (3), we know that soliton' width is determined by the parameter $C_2(t)$. When $C_2(t) > 0$, its width will be compressed and vice versa; when $C_2(t)$ vanishes, its width will keep invariant. If the evolution pattern of soliton is chosen, the expression of $C_2(t)$ is made certain. Then $R(t) = g\exp[\int G(t) - 4C_2(t)\,dt]$ can tells us how to manage Feshbach resonance, and $4C_2^2(t) + \frac{dC_2(t)}{dt} = M(t)$ presents explicit way to manage the trap potential, which can be realized through modulating the ratio of axial oscillation to radial oscillation. Therefore, we can know how to adjust the related operations to control the evolution of soliton's width.

(2) From Eq. (4), it is known that the peak of soliton is determined by the parameters $C_2(t)$ and $G(t)$. Considering the two factors relate with trap potential and complex potential, we infer that the peak and width of the bright soliton can be controlled well by adjusting the experiment parameters, including the ratio of axial oscillation to radial oscillation and exchanging atoms between the condensates and thermal cloud. Especially, when $G(t) = -4C_2(t)$, the peak of soliton will become unchanged as a constant $\frac{4\alpha^2}{g}$. For an example, when $M(t) = \lambda^2/4$, then $C_2(t)$ can be $\lambda/4$, if $G(t) = -\lambda$, then the peak of soliton will be a constant, which agrees with the result in [21]. It is well known that soliton comes from the balance between nonlinearity and dispersion effects. However, the dispersion is unchanged here, the condition we find is just the balance of nonlinearity and gain term for constant peak.

(3) From Eq. (5), one can know which factors affect the motion of soliton. Moreover, one can calculate soliton's velocity, and acceleration as following

$$v_c(t) = 4\left[\int \frac{f(t)}{2}e^{\int 4C_2(t)\,dt}\,dt + \beta\right]\exp\left[\int -4C_2(t)\,dt\right]$$

[1] The Lax-pair we use is

$$\partial_x\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = \begin{pmatrix}\zeta & \sqrt{g}Q\\ -\sqrt{g}\bar{Q} & \zeta\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix},$$

$$\partial_t\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = \begin{pmatrix}A & B\\ C & -A\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix},$$

where

$$A = 2i\zeta^2 - 4C_2(t)x\zeta + ig|Q|^2 + if(t)x/2,$$

$$B = 2i\sqrt{g}\zeta Q + i\sqrt{g}Q_x - 4\sqrt{g}C_2(t)xQ,$$

$$C = -2i\sqrt{g}\zeta\bar{Q} + i\sqrt{g}\bar{Q}_x + 4\sqrt{g}C_2(t)x\bar{Q},$$

$$\zeta_t = -4C_2(t)\zeta + if(t)/2.$$

and the overbar denotes the complex conjugate.