Physics Letters A 379 (2015) 1510–1519

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Rational solutions to two- and one-dimensional multicomponent Yajima–Oikawa systems

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ARTICLE INFO

Article history:
Received 10 December 2014
Received in revised form 24 February 2015
Accepted 24 February 2015
Available online 27 February 2015
Communicated by C.R. Doering

Keywords:
Multicomponent Yajima–Oikawa system
Bilinear method
Rational solution
Two-dimensional dark rogue wave

ABSTRACT

Exact explicit rational solutions of two- and one-dimensional multicomponent Yajima–Oikawa (YO) systems, which contain multi-short-wave components and single long-wave one, are presented by using the bilinear method. For two-dimensional system, the fundamental rational solution first describes the localized lumps, which have three different patterns: bright, intermediate and dark states. Then, rogue waves can be obtained under certain parameter conditions and their behaviors are also classified to above three patterns with different definition. It is shown that the simplest (fundamental) rogue waves are line localized waves which arise from the constant background with a line profile and then disappear into the constant background again. In particular, two-dimensional intermediate and dark counterparts of rogue wave are found with the different parameter requirements. We demonstrate that multirogue waves describe the interaction of several fundamental rogue waves, in which interesting curvy wave patterns appear in the intermediate times. Different curvy wave patterns form in the interaction of different types fundamental rogue waves. Higher-order rogue waves exhibit the dynamic behaviors that the wave structures start from lump and then retreat back to it, and this transient wave possesses the patterns such as parabolas. Furthermore, different states of higher-order rogue wave result in completely distinguishing lumps and parabolas. Moreover, one-dimensional rogue wave solutions with three states are constructed through the further reduction. Specifically, higher-order rogue wave in one-dimensional case is derived under the parameter constraints.

1. Introduction

Rogue wave phenomena that "appear from nowhere and disappear without a trace [1]", have recently become one of the most active and important research areas on both experimental observations and theoretical analysis, since it exists in various different fields, including ocean [2], optical systems [3–5], Bose–Einstein condensates [6,7], superfluids [8], capillary waves [9], atmosphere [10], plasma [11,12] and even in finance [13]. From the mathematical description, rational solutions play a key role in the interpretation of the mechanisms underlying the formation and dynamics of rogue waves. The first-order and most fundamental rational solution for nonlinear Schrödinger (NLS) equation was discovered by Peregrine [14]. Such a solution has the peculiarity of being localized in both space and time, and its maximum amplitude reaches three times the constant background. The hierarchy of higher-order rational solutions has been found [15–24], in particular, in the framework of the integrable 1D NLS equation. These higher-order waves were also localized in both coordinates, and could exhibit higher peak amplitudes or multiple intensity peaks.

Recently, apart from the NLS equation, exact rogue wave solutions have been explored in a variety of nonlinear integrable systems such as the Hirota equation [25,26], the Sasa–Satsuma equation [27,28] and the derivative NLS equation [29–31]. More importantly, the relevant studies were also extended to coupled systems which usually involve more than one component [7,32–47]. It was shown that compared with uncoupled systems, vector rogue wave solutions exhibit some novel structures such as dark rogue wave. In Refs. [32–34], analytical rational solutions for the coupled NLS system allowed not only general vector Peregrine soliton but also bright- and dark-rogue waves.

Moreover, the two-dimensional analogue of rogue wave, expressed by more complicated rational form, has been recently reported in the Davey–Stewartson (DS) equation [48,49] and Kadomtsev–Petviashvili-I equation [20,50]. In two kinds of DS sys-

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http://dx.doi.org/10.1016/j.physleta.2015.02.040
0375-9601/

tems [48,49], the fundamental rogue waves are line rogue waves which arise from the constant background and then retreat back to the constant background again. More general rational solutions were divided into two categories: multi-rogue waves and higher order ones. Multi-rogue waves describe the interaction between individual fundamental rogue waves, whereas higher order rogue waves exhibit different dynamics such as the wavepacket rising from the constant background but not decaying back to it. Therefore, a natural motivation is to investigate rational solutions in two-dimensional multicomponent system. Specifically, it is reasonable to expect the appearance of a two-dimensional dark rogue wave counterpart, which, to the best our knowledge, was never reported before.

Coming back to the one-dimensional case, rogue waves were usually obtained from homoclinic solutions by taking certain limits [25–27,29,31,37]. Indeed, most of literature devoted to the explicit expressions of rational solutions still resulted from the related homoclinic ones. The construction of higher-dimensional rational solutions may provide an alternative method for finding lower-dimensional rogue wave through dimension reduction directly [48,49]. In other words, one can generate the above rational solutions of lower-dimensional models from higher-dimensional ones with the parameter constraints. Application of reduction method to clarify the rational solution's relation between two different dimensions is also the aim of the present work.

In this paper, we focus on the two-dimensional multicomponent Yajima–Oikawa (YO) system, or the so-called 2D coupled long-wave–short-wave resonance interaction system in which it comprises multi short-wave components and a single long-wave component [51–56]. The long-wave–short-wave resonance interaction is a fascinating physical process in which a resonant interaction takes place between a weakly dispersive long-wave and a short-wave packet when the phase velocity of the former exactly or almost matches the group velocity of the latter. This phenomenon has been predicted in plasma physics [57,58], nonlinear optics [59,60] and hydrodynamics [61–63]. The rogue wave solutions to the 1D YO system had recently been derived by using Hirota bilinear method [64] and Darboux transformation [39,40]. A special note of importance is that the coupled dark- and bright-field counterparts of the Peregrine soliton were demonstrated in [38–40].

The plan of the paper is as follows. In Section 2, we present exact and explicit rational solution for the two-dimensional multicomponent YO system by using the bilinear method. In Section 3, dynamics of two-dimensional rational solution including fundamental lumps and general (multi- and higher-order) rogue waves are discussed in detail. The one-dimensional rogue wave solution is derived through the further reduction and its dynamics are studied in Section 4. The conclusion is given in the last section.

2. Explicit rational solution in the determinant form

The two-dimensional multicomponent YO system:

$$\mathrm{i}(S_t^{(\ell)}+S_y^{(\ell)})-S_{xx}^{(\ell)}+LS^{(\ell)}=0,\quad \ell=1,2,\cdots,M, \tag{1a}$$

$$L_t=2\sum_{\ell=1}^{M}\sigma_\ell|S^{(\ell)}|_x^2, \tag{1b}$$

where $\sigma_\ell=\pm1$, $S^{(\ell)}$ and L indicate the ℓth short-wave and long-wave components, respectively. When the wave propagation is independent of y coordinate Eq. (1) is reduced to the one-dimensional multicomponent YO system. By the dependent variable transformation:

$$S^{(\ell)}=G_0^{(\ell)}\frac{G^{(\ell)}}{F},\quad L=h-2\frac{\partial^2}{\partial x^2}\log F, \tag{2}$$

where $G_0^{(\ell)}=\rho_\ell\exp[\mathrm{i}(\alpha_\ell x+\beta_\ell y+\gamma_\ell t)]$, $\gamma_\ell=h-\beta_\ell+\alpha_\ell^2$ and $\alpha_\ell,\beta_\ell,\rho_\ell$ and h are real parameters for $\ell=1,2,\cdots,M$, the two-dimensional YO system can be cast into the bilinear form,

$$[\mathrm{i}(D_t+D_y-2\alpha_\ell D_x)-D_x^2]G^{(\ell)}\cdot F=0, \tag{3a}$$

$$D_tD_xF\cdot F-2\sum_{\ell=1}^{M}\sigma_\ell\rho_\ell^2F^2+2\sum_{\ell=1}^{M}\sigma_\ell\rho_\ell^2G^{(\ell)}G^{(\ell)*}=0, \tag{3b}$$

where F is a real variable, $G^{(\ell)}$ are complex variables, $*$ denotes the complex conjugation and D is Hirota's bilinear differential operator.

Theorem 1. *The two-dimensional multicomponent YO system has rational solution (2) with F and $G^{(\ell)}$ given by $N\times N$ determinants*

$$F=\tau'(n)\Big|_{n=0},\quad G^{(\ell)}=\tau'(n^{(\ell)}+1)\Big|_{n=0}, \tag{4}$$

where $(n)\equiv(n^{(1)},n^{(2)},\cdots,n^{(M)})$, $(n^{(\ell)}\pm1)\equiv(n^{(1)},n^{(2)},\cdots,n^{(\ell)}\pm1,\cdots,n^{(M)})$ and $n=0$ represents $n^{(1)}=n^{(2)}=\cdots n^{(\ell)}\cdots=n^{(M)}=0$, $\tau'(n)=\det_{\le i,j\le N}\left(T'_{i,j}(n)\right)$ and the matrix elements are defined by

$$T'_{i,j}(n)=\prod_{\ell=1}^{M}(-\frac{p_i-\mathrm{i}\alpha_\ell}{p_j^*+\mathrm{i}\alpha_\ell})^{n^{(\ell)}}\mathcal{A}_{i,j}\frac{1}{p_i+p_j^*}. \tag{5}$$

Here the operator $\mathcal{A}_{i,j}=\sum_{k=0}^{n_i}c_{ik}(\partial_{p_i}+\xi_i'+\sum_{\ell=1}^{M}\frac{n^{(\ell)}}{p_i-\mathrm{i}\alpha_\ell})^{n_i-k}\times\sum_{l=0}^{n_j}c_{jl}^*(\partial_{p_j^*}+\xi_j'^*-\sum_{\ell=1}^{M}\frac{n^{(\ell)}}{p_j^*+\mathrm{i}\alpha_\ell})^{n_j-l}$ and

$$\xi_i'=-\sum_{\ell=1}^{M}\frac{\sigma_\ell\rho_\ell^2(t-y)}{(p_i-\mathrm{i}\alpha_\ell)^2}+x-2\mathrm{i}p_iy, \tag{6}$$

where p_i and c_{ik} are arbitrary complex constants, and n_i is an arbitrary positive integer.

The proof of this theorem is given in Appendix A. It is emphasized that these rational solutions can also be expressed in term of Schur polynomials as shown in [48,49]. From the appendix in [48,49], one can know that the nonsingularity of rational solutions exists if the real parts of wave numbers p_i $(1\leqslant i\leqslant N)$ are all positive or negative.

3. Rational solutions for two-dimensional YO system

In this section, we present the dynamics analysis of rational solutions to two-dimensional YO system in detail.

3.1. Fundamental rational solutions

As the simplest rational solution, one-rational solution of first order is given by taking $N=1$ and $n_1=1$,

$$\begin{aligned}F&=\sum_{k=0}^{1}c_{1k}(\partial_{p_1}+\xi_1')^{1-k}\sum_{l=0}^{1}c_{1l}^*(\partial_{p_1^*}+\xi_1'^*)^{1-l}\frac{1}{p_1+p_1^*}\\&=(\partial_{p_1}+\xi_1'+c_{11})(\partial_{p_1^*}+\xi_1'^*+c_{11}^*)\frac{1}{p_1+p_1^*}\\&=\frac{1}{p_1+p_1^*}\left[\left(\xi_1'-\frac{1}{p_1+p_1^*}+c_{11}\right)\left(\xi_1'^*-\frac{1}{p_1+p_1^*}+c_{11}^*\right)\right.\\&\quad\left.+\frac{1}{(p_1+p_1^*)^2}\right],\end{aligned} \tag{7}$$