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Rotational elastic waves in double wall tube

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ABSTRACT

We describe the double wall tube with cylindrical dislocation in the framework of the geometric theory of defects. The induced metric is found. The dispersion relation is obtained for the propagation of rotational elastic waves in the double wall tube.

1. Introduction

Ideal crystals are absent in nature, and most of their physical properties, such as plasticity, melting, growth, etc., are defined by defects of the crystalline structure. Therefore, a study of defects is a topical scientific question of importance for applications in the first place. At present, a fundamental theory of defects is absent in spite of the existence of dozens of monographs and thousands of articles.

One of the most promising approaches to the theory of defects is based on Riemann–Cartan geometry, which involves nontrivial metric and torsion. In this approach, a crystal is considered as a continuous elastic medium with a spin structure. If the displacement vector field is a smooth function, then there are only elastic stresses corresponding to diffeomorphisms of the Euclidean space. If the displacement vector field has discontinuities, then we are saying that there are defects in the elastic structure. Defects in the elastic structure are called dislocations and lead to the appearance of nontrivial geometry. Precisely, they correspond to a nonzero torsion tensor, equal to the surface density of the Burgers vector. Defects in the spin structure are called disclinations. They correspond to nonzero curvature tensor, curvature tensor being the surface density of the Frank vector.

The idea to relate torsion to dislocations appeared in the 1950s [1–4]. This approach is still being successfully developed (note reviews [5–11]), and is often called the gauge theory of dislocations.

Some time ago we proposed the geometrical theory of defects [12–14]. Our approach is essentially different from others in two respects. Firstly, we do not have the displacement and rotational angle vector fields as independent variables because, in general, they are not continuous. Instead, the triad field and $\mathbb{SO}(3)$-connection are considered as independent variables. If defects are absent, then the triad and $\mathbb{SO}(3)$-connection reduce to partial derivatives of the displacement and rotational angle vector fields. In this case the latter can be reconstructed. Secondly, the set of equilibrium equations is different. We proposed purely geometric set which coincides with that of Euclidean three dimensional gravity with torsion. The nonlinear elasticity equations and principal chiral $\mathbb{SO}(3)$ model for the spin structure enter the model through the elastic and Lorentz gauge conditions [14–16] which allow to reconstruct the displacement and rotational angle vector fields in the absence of dislocations in full agreement with classical models.

The advantage of the geometric theory of defects is that it allows one to describe single defects as well as their continuous distributions.

In the present paper, we consider propagation of rotational elastic waves in double wall tube with cylindrical dislocation. This defect was first described in [17]. The Schrödinger equation for the double wall tube was solved in [18] and applied to double wall nanotubes. A similar problem was also solved for the cylindrical waveguide with wedge dislocation [19].

1.1. Double wall tube

Let us describe double wall tube with cylindrical dislocation in the framework of the geometric theory of defects.

We consider cylindrical coordinates $\{x^\mu\} = \{r, \varphi, z\}$, $\mu = 1, 2, 3$ in tree dimensional Euclidean space $\mathbb{R}^3$. Let there be two thick tubes $r_0 \le r \le r_1$ and $r_2 \le r \le r_3$ of elastic media, each axis coinciding with the z axis. We suppose that $r_0 < r_1 < r_2 < r_3$ (see Fig. 1(a), where a section $z = \text{const}$ is shown). Now we make one tube with the inside cylindrical dislocation in the following manner. We stretch symmetrically the inner tube and compress the outer one. Then glue together the external surface of the inner tube with the internal surface of the outer tube. Afterwards the media comes to some equilibrium state. Due to rotational and translational symmetry we obtain one tube $r_{in} \le r \le r_{ex}$ with the

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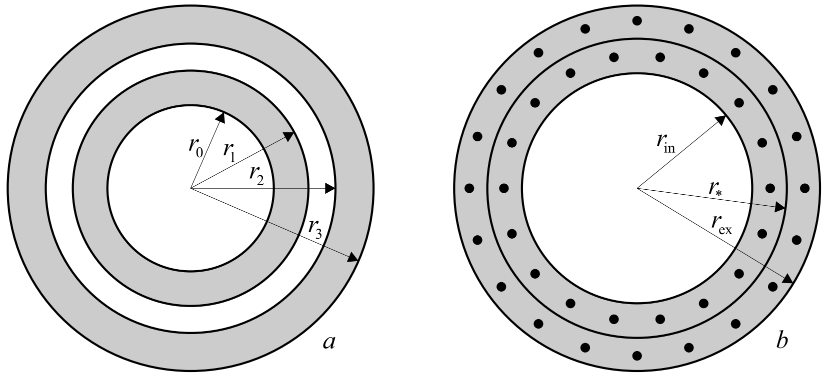

Fig. 1. Section $z =$ const of double wall tube before (a) and after (b) defect creation.

axis which coincides with the z axis (see Fig. 1(b)). Radii of cylinders constituting tube surfaces are mapped as follows

$$r_0 \mapsto r_{\rm in}, \qquad r_1, r_2 \mapsto r_*, \qquad r_3 \mapsto r_{\rm ex}.$$

The gluing is performed along the cylinder r_*, and there is cylindrical defect (dislocation) because part of the media between tubes is removed.

The obtained double wall tube with cylindrical dislocation is rotationally and translationally symmetric.

The constructed model of the tube with cylindrical dislocation can be considered as continuous model of double wall nanotube (for a general review, see [20–22]). Consider double wall nanotube having two atomic layers. Suppose the inner layer has 18 and outer layer has 20 atoms which are shown in Fig. 1(b) by points. Natural length measure here is the interatomic distance. Then the length of a circle has a jump when one goes from inner to outer layer. In the geometric theory of defects, it means that the metric component $g_{\varphi\varphi}$ is not continuous in cylindrical coordinates. The corresponding model will be described below.

To find radii $r_{\rm in}$, r_*, and $r_{\rm ex}$ we have to solve the classical elasticity problem.

Let us define the displacement vector field by $u^i(x)$, $i = 1, 2, 3$,

$$y^i \mapsto x^i = y^i + u^i(x), \tag{1}$$

where y^i and x^i are coordinates of a point before and after deformation respectively. We consider the displacement field as a vector function on points of media after deformation and gluing. This is more adequate because the resulting media after gluing is a connected manifold (before the gluing procedure, each tube represents a connected component). In equilibrium state, the vector displacement field satisfies the equation

$$(1 - 2\sigma)\triangle u_i + \partial_i \partial_j u^j = 0, \tag{2}$$

where σ is the Poisson ratio and $\triangle$ is the Laplacian. For convenience, we consider components of the displacement vector field with respect to the orthonormal basis

$$u = u^{\hat r} e_{\hat r} + u^{\hat\varphi} e_{\hat\varphi} + u^{\hat z} e_{\hat z},$$

where

$$e_{\hat r} = \partial_r, \qquad e_{\hat\varphi} = \frac{1}{r}\partial_\varphi, \qquad e_{\hat z} = \partial_z.$$

We denote indices with respect to the orthonormal basis by hat:

$$\{i\} = \{\hat r, \hat\varphi, \hat z\}, \qquad \{\mu\} = \{r, \varphi, z\}.$$

The Latin indices referred to an orthonormal basis are raised and lowered by Kronecker symbol: $u_i := u^j \delta_{ji}$.

The divergence and Laplacian have the following form in cylindrical coordinates

$$\begin{aligned}
\nabla_i u^i &= \frac{1}{r}\partial_r(ru^{\hat r}) + \frac{1}{r}\partial_\varphi u^{\hat\varphi} + \partial_z u^{\hat z},\\
\triangle u_{\hat r} &= \frac{1}{r}\partial_r(r\partial_r u_{\hat r}) + \frac{1}{r^2}\partial^2_{\varphi\varphi}u_{\hat r} + \partial^2_{zz}u_{\hat r} - \frac{1}{r^2}u_{\hat r} - \frac{2}{r^2}\partial_\varphi u_{\hat\varphi},\\
\triangle u_{\hat\varphi} &= \frac{1}{r}\partial_r(r\partial_r u_{\hat\varphi}) + \frac{1}{r^2}\partial^2_{\varphi\varphi}u_{\hat\varphi} + \partial^2_{zz}u_{\hat\varphi} - \frac{1}{r^2}u_{\hat\varphi} + \frac{2}{r^2}\partial_\varphi u_{\hat r},\\
\triangle u_{\hat z} &= \frac{1}{r}\partial_r(r\partial_r u_{\hat z}) + \frac{1}{r^2}\partial^2_{\varphi\varphi}u_{\hat z} + \partial^2_{zz}u_{\hat z}.
\end{aligned} \tag{3}$$

From the symmetry of the problem, we deduce that only radial component of the displacement field differs from zero, and it does not depend on the angle φ and z coordinates:

$$\{u^i\} = \{u^{\hat r} := u(r), u^{\hat\varphi} = 0, u^{\hat z} = 0\}.$$

Eq. (2) for zero $u_{\hat\varphi}$ and $u_{\hat z}$ components is automatically satisfied. It is easy to check that the radial derivative of the divergence,

$$\partial_{\hat r}\partial_j u^j = \partial_r\left(\frac{1}{r}\partial_r(ru)\right) = \partial^2_{rr}u + \frac{1}{r}\partial_r u - \frac{1}{r^2}u,$$

coincides with the Laplacian

$$\triangle u_{\hat r} = \frac{1}{r}\partial_r(r\partial_r u) - \frac{1}{r^2}u = \partial^2_{rr}u + \frac{1}{r}\partial_r u - \frac{1}{r^2}u.$$

Therefore the radial component of Eq. (2) takes the form

$$\partial_r\left(\frac{1}{r}\partial_r(ru)\right) = 0. \tag{4}$$

A general solution of this equation depends on two integration constants:

$$u = c_1 r + \frac{c_2}{r}, \qquad c_{1,2} = \text{const}.$$

Note that the equilibrium equation (4) does not depend on the Poisson ratio σ. This means that the cylindrical dislocation is the geometrical defect.

Boundary conditions have to be imposed to fix the integration constants. Let us introduce a notation for inner and outer tubes:

$$u = \begin{cases} u_{\rm in}, & r_{\rm in} \le r \le r_*,\\ u_{\rm ex}, & r_* \le r \le r_{\rm ex}.\end{cases}$$

Now boundary conditions are to be imposed. We assume that the surface of two wall nanotube is free, i.e. the deformation tensor is zero on the boundary:

$$\left.\frac{du_{\rm in}}{dr}\right|_{r=r_{\rm in}} = 0, \qquad \left.\frac{du_{\rm ex}}{dr}\right|_{r=r_{\rm ex}} = 0. \tag{5}$$