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Spacetime structure and asymmetric metric from the premetric formulation of electromagnetism

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ABSTRACT

We address the issue of spacetime structure determined empirically from the premetric formulation of electromagnetism and explore the role of skewons in the construction of spacetime metric. Type II skewon part is not constrained in the first order. In the second order it induces birefringence and is constrained to $\sim 10^{-19}$. However, an additional nonmetric induced second-order contribution to the core-metric principal part makes it nonbirefringent. This second-order contribution is just the extra piece to the core-metric principal constitutive tensor induced by the antisymmetric part of the asymmetric metric which is nonbirefringent. The antisymmetric metric induced constitutive tensor has a pseudoscalar part. The variation of this part is constrained by observation on cosmic polarization rotation to <0.03, and gives one constraint on the 6-degree-of-freedom antisymmetric metric.

1. Introduction – premetric formulation of electrodynamics

In the historical development, special relativity arose from the invariance of Maxwell equations under Lorentz transformation. In 1908, Minkowski [1] further put it into 4-dimensional geometric form with a metric invariant under Lorentz transformation. The use of metric as dynamical gravitational potential [2] and the employment of Einstein Equivalence Principle for coupling gravity to matter [3] are two important cornerstones to build general relativity [2,4]. In putting Maxwell equations into a form compatible with general relativity, Einstein in 1916 noticed that the equations can be formulated in a form independent of the metric gravitational potential [5,6]. Weyl [7], Murnaghan [8], Kottler [9] and Cartan [10] further developed and clarified this resourceful approach.

Maxwell equations for macroscopic/spacetime electrodynamics in terms of independently measurable field strength F_{kl} ($\boldsymbol{E}$, $\boldsymbol{B}$) and excitation (density with weight +1) H^{ij} ($\boldsymbol{D}$, $\boldsymbol{H}$) do not need metric as primitive concept (see, e.g., Hehl and Obukhov [11]):

$$H^{ij}{}_{,j} = -4\pi J^i, \tag{1a}$$

$$e^{ijkl} F_{jk,l} = 0, \tag{1b}$$

with J^k the charge 4-current density and e^{ijkl} the completely antisymmetric tensor density of weight +1 ($e^{0123} = 1$). We use units with the nominal light velocity c equal to 1. To complete this set of equations, a constitutive relation is needed between the excitation and the field:

$$H^{ij} = \chi^{ijkl} F_{kl}. \tag{2}$$

Both H^{ij} and F_{kl} are antisymmetric, hence χ^{ijkl} must be antisymmetric in i and j, and in k and l. Hence the constitutive tensor density χ^{ijkl} (with weight +1) has 36 independent components, and can be uniquely decomposed into principal part (P), skewon part (Sk) and axion part (Ax) as given in [11,12]:

$$\chi^{ijkl} = {}^{(\mathrm{P})}\chi^{ijkl} + {}^{(\mathrm{Sk})}\chi^{ijkl} + {}^{(\mathrm{Ax})}\chi^{ijkl}$$
$$\left(\chi^{ijkl} = -\chi^{jikl} = -\chi^{ijlk}\right) \tag{3}$$

with

$${}^{(\mathrm{P})}\chi^{ijkl} = (1/6)\big[2\big(\chi^{ijkl} + \chi^{klij}\big) - \big(\chi^{iklj} + \chi^{ljik}\big) - \big(\chi^{iljk} + \chi^{jkil}\big)\big], \tag{4a}$$

$${}^{(\mathrm{Ax})}\chi^{ijkl} = \chi^{[ijkl]} = \varphi e^{ijkl}, \tag{4b}$$

$${}^{(\mathrm{Sk})}\chi^{ijkl} = (1/2)\big(\chi^{ijkl} - \chi^{klij}\big). \tag{4c}$$

The principal part has 20 degrees of freedom. The axion part has one degree of freedom. The Hehl–Obukhov–Rubilar skewon part (4c) can be represented as

$${}^{(\mathrm{Sk})}\chi^{ijkl} = e^{ijmk} S_m{}^l - e^{ijml} S_m{}^k, \tag{5}$$

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with $S_m{}^n$ a traceless tensor of 15 independent degrees of freedom [11].

There are two equivalent definitions of constitutive tensor which are useful in various discussions (see, e.g., Hehl and Obukhov [11]). The first one is to take a dual on the first 2 indices of χ^{ijkl}:

$$\kappa_{ij}{}^{kl} \equiv (1/2)\underline{e}_{ijmn}\chi^{mnkl}, \tag{6}$$

where $\underline{e}_{ijmn}$ is the completely antisymmetric tensor density of weight -1 with $\underline{e}_{0123} = 1$. Since e_{ijmn} is a tensor density of weight -1 and χ^{mnkl} a tensor density of weight $+1$, $\kappa_{ij}{}^{kl}$ is a (twisted) tensor. From (6), we have

$$\chi^{mnkl} = (1/2)e^{ijmn}\kappa_{ij}{}^{kl}. \tag{7}$$

With this definition of constitutive tensor $\kappa_{ij}{}^{kl}$, the constitutive relation (2) becomes

$$^*H_{ij} = \kappa_{ij}{}^{kl}F_{kl}, \tag{8}$$

where $^*H_{ij}$ is the dual of H^{ij}, i.e.

$$^*H_{ij} \equiv (1/2)\underline{e}_{ijmn}H^{mn}. \tag{9}$$

The second equivalent definition of the constitutive tensor is to use a 6×6 matrix representation $\kappa_I{}^J$. Since $\kappa_{ij}{}^{kl}$ is nonzero only when the antisymmetric pairs of indices (ij) and (kl) have values (01), (02), (03), (23), (31), (12), the index pairs can be enumerated by capital letters $I, J, \ldots$ from 1 to 6 to obtain $\kappa_I{}^J$ $(\equiv \kappa_{ij}{}^{kl})$. With the relabeling, $F_{ij} \to F_I$, $H^{ij} \to H^I$, $\underline{e}_{ijmn} \to \underline{e}_{IJ}$, $e^{ijmn} \to e^{IJ}$. We have $F_I = (\boldsymbol{E}, -\boldsymbol{B})$ and $(^*H)_I = (-\boldsymbol{H}, -\boldsymbol{D})$. $\underline{e}_{IJ}$ and e^{IJ} can be expressed in matrix form as

$$\underline{e}_{IJ} = e^{IJ} = \begin{pmatrix} 0 & \mathbf{I}_3 \\ \mathbf{I}_3 & 0 \end{pmatrix}, \tag{10}$$

where $\mathbf{I}_3$ is the 3×3 unit matrix. In terms of this definition, the constitutive relation (8) becomes

$$^*H_I = 2\kappa_I{}^J F_J, \tag{11}$$

where $^*H_I \equiv {}^*H_{ij} = e_{IJ}H^J$. The axion part $^{(\mathrm{Ax})}\chi^{ijkl}$ $(= \varphi e^{ijkl})$ now corresponds to

$$^{(\mathrm{Ax})}\kappa_I{}^J = \varphi \begin{pmatrix} \mathbf{I}_3 & 0 \\ 0 & \mathbf{I}_3 \end{pmatrix} = \varphi \mathbf{I}_6, \tag{12}$$

where $\mathbf{I}_6$ is the 6×6 unit matrix. The principal part and the axion part of the constitutive tensor all satisfy the following equation (the skewonless condition):

$$e^{KJ}\kappa_J{}^I = e^{IJ}\kappa_J{}^K. \tag{13}$$

In Section 2, we discuss and study the empirical construction of spacetime structure from premetric electrodynamics in the skewonless case. In Section 3, we discuss the empirical foundation of the closure relation. In Section 4, we study the constraints on Type II skewon field to second order. Although the constitutive tensor with the metric principal part and Type II skewon part is birefringent in the second order, an added 2nd order nonmetric piece to the metric principal part would make the constitutive tensor nonbirefringent. This added part turns out to be just the extra piece in the principal part of an asymmetric-metric induced constitutive tensor. In Section 5, we turn to the study of dispersion relation of an asymmetric-metric induced constitutive tensor. In Section 6, we study the experimental/observational constraints on the asymmetric-metric induced spacetime constitutive tensor. In Section 7, we present an outlook and a few discussions.

2. Construction of spacetime structure from premetric electrodynamics in the skewonless case

The first issue here is that how to (with what conditions can we) reach a metric or, owing to conformal invariance, how to reach a Riemannian light cone (a core metric up to conformal invariance) from the constitutive tensor. This issue has been studied rather thoroughly in the skewonless case, i.e. in the case that χ^{ijkl} is symmetric under the exchange of the index pairs ij and kl (in terms of the 6×6 matrix representation $\kappa_I{}^J$, Eq. (13) is satisfied) and has 20 principal components and 1 axionic component. In this case, the Maxwell equations can be derived from the Lagrangian L $(= L_I{}^{(\mathrm{EM})} + L_I{}^{(\mathrm{EM\text{-}P})})$ with the electromagnetic field Lagrangian $L_I{}^{(\mathrm{EM})}$ and the field–current interaction Lagrangian $L_I{}^{(\mathrm{EM\text{-}P})}$ given by

$$L_I{}^{(\mathrm{EM})} = -\big(1/(8\pi)\big)H^{ij}F_{ij} = -\big(1/(8\pi)\big)\chi^{ijkl}F_{ij}F_{kl}, \tag{14}$$

$$L_I{}^{(\mathrm{EM\text{-}P})} = -A_k J^k, \tag{15}$$

with $\chi^{ijkl} = -\chi^{jikl}$ a tensor density of the gravitational fields or matter fields to be investigated, $F_{ij} \equiv A_{j,i} - A_{i,j}$ the electromagnetic field strength tensor, A_i the electromagnetic 4-potential guaranteed by the second Maxwell equation (1b), and comma denoting partial derivation. We note that only the part of χ^{ijkl} which is symmetric under the interchange of index pairs ij and kl contributes to the Lagrangian, i.e. skewon part does not contribute and we assume it is absent in this section. Three conditions have been studied for this symmetric constitutive tensor:

(i) The closure condition: Toupin [13], Schönberg [14], and Jadczyk [15] have investigated this approach. The closure condition on the skewonless constitutive tensor is

$$\kappa\kappa = \big(\kappa_I{}^J\kappa_J{}^K\big) = (1/6)\,\mathrm{tr}(\kappa\kappa)\mathbf{I}_6. \tag{16}$$

With this closure condition, it has been shown that the constitutive tensor must be metric with a dilatonic degree of freedom ψ, i.e.,

$$\chi^{ijkl} = (-h)^{1/2}\big[(1/2)h^{ik}h^{jl} - (1/2)h^{il}h^{kj}\big]\psi, \tag{17}$$

where h^{ij} is a symmetric metric with inverse h_{ij}, $h = \det(h_{ij})$, and ψ is a scalar (dilaton) degree of freedom.

(ii) The Galileo weak equivalence principle: In the 1970s, we used Galileo Equivalence Principle and derived its consequences for an electromagnetic system with the particle Lagrangian $L_I{}^{(\mathrm{P})}$ [16,17]:

$$L_I{}^{(\mathrm{P})} = -\sum_I m_I(ds_I)/(dt)\delta(\boldsymbol{x} - \boldsymbol{x}_I). \tag{18}$$

Here m_I is the mass of the Ith (charged) particle and s_I its 4-line element from the metric g_{ij}. The result is that the constitutive tensor density χ^{ijkl} and the particle metric must satisfy the following relation:

$$\chi^{ijkl} = (-g)^{1/2}\big[(1/2)g^{ik}g^{jl} - (1/2)g^{il}g^{kj}\big] + \varphi e^{ijkl}, \tag{19}$$

where g^{ij} is the inverse of the particle metric g_{ij}, $g = \det(g_{ij})$, and φ is a pseudoscalar (axion) degree of freedom in the relation. Since it is well-known that the axion degree of freedom does not affect the propagation of light in the lowest eikonal approximation [11,18–24], the particle metric g^{ij} (or g_{ij}) also generates the light cone for electromagnetic wave propagation in this approximation. We note in passing that from (19) the Galileo Equivalence Principle sets the dilaton field to 1 (constant) and does not allow its variation in the χ–g framework.

(iii) The nonbirefringence condition: The third approach used empirical observations/experiments to constrain the constitutive