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Phenomenological description of anisotropy effects in some ferromagnetic superconductors

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ABSTRACT

We study phenomenologically the role of anisotropy in ferromagnetic superconductors UGe_2, URhGe, and UCoGe for the description of their phase diagrams. We use the Ginzburg–Landau free energy in its uniform form as we will consider only spatially independent solutions. This is an expansion of previously derived results where the effect of Cooper-pair and crystal anisotropies is not taken into account. The three compounds are separately discussed with the special stress on UGe_2. The main effect comes from the strong uniaxial anisotropy of magnetization while the anisotropy of Cooper pairs and crystal anisotropy only slightly change the phase diagram in the vicinity of Curie temperature. The limitations of this approach are also discussed.

1. Introduction

The discovery of ferromagnetic superconductors UGe_2 [1], URuGe [2], UCoGe [3], in which ferromagnetic ordering coexists with superconductivity, has given a new trend in understanding of unconventional superconductivity. The pressure–temperature phase diagrams of these compounds differ, but the common feature is that the superconductivity occurs in the domain of ferromagnetic phase and the superconducting transition temperature T_s is lower than the Curie temperature, T_c. UGe_2 orders ferromagnetically at relatively high Curie temperature of 53 K and superconductivity appears upon the application of pressure of about 1 GPa, and at low temperature <1 K. The increase of pressure to the critical value $P_c = 1.5$ GPa results in disappearance of both ferromagnetic and superconducting orders.

URuGe and UCoGe are weaker ferromagnets with T_c of 9.5 K and 3 K, respectively and the superconducting phase appears at ambient pressure as well. For URuGe the increase of pressure leads to the collapse of superconductivity at about 4 GPa, while for UCoGe the phase transition line gradually grows reaching maximum at 1.1 GPa, where the ferromagnetic order collapses and superconductivity persists also in the paramagnetic region. All three uranium compounds have orthorhombic crystal structure with highly anisotropic magnetic moment of Ising type. For detailed experimental presentation of ferromagnetic superconductors, see, for example, the recent review [4].

It is commonly accepted that 5f electrons of uranium atoms are responsible for both ferromagnetic and superconducting orders. In the presence of magnetization, the ferromagnetic exchange field is expected to rule out spin-singlet Cooper pairing and unconventional superconductivity of p-type, mediated through some magnetic mechanism is considered as the most likely. The experimental discovery of huge upper critical field in URhGe and UCoGe also confirms the triplet pairing because the Pauli paramagnetic effect characteristic of spin singlet pairing is absent there, see, for example [5,20] and the papers cited therein.

The coexistence of itinerant ferromagnetism and superconductivity is theoretically proposed in [6] and the main idea is that the exchange of longitudinal spin fluctuations may lead to the triplet pairing in weak itinerant ferromagnets. There are theoretical considerations supported by experimental results that the scenario of spin-triplet superconductivity induced by longitudinal ferromagnetic spin fluctuations can be realized in UCoGe [7,8]. A recent theoretical trend generalizes the phenomenological Ginzburg–Landau approaches on the basis of lattice model for the description of both Meissner and inhomogeneous states of ferromagnetic superconductivity, see [9] and the review [10].

The anisotropic properties of superconductivity in ferromagnetic superconductors are vastly studied experimentally, especially the anisotropic properties of upper critical field [4]. Hattori *et al.* [7] claimed that the ferromagnetic fluctuations with Ising

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type anisotropy are responsible for the appearance of homogeneous ferromagnetic superconductivity in UCoGe.

Here we will study phenomenologically the role of magnetic, crystal and Cooper-pair anisotropy on the phase diagram and possible phases using the previously derived Ginzburg–Landau free energy [11].

2. Landau free energy

We will consider only the Meissner phases – pure superconductors and phases of coexistence of ferromagnetism and superconductivity in the absence of external magnetic field. In earlier papers [11] we did not take in account the Cooper-pair and crystal anisotropies as the purpose there was to model phenomenologically the P–T phase diagram of UGe_2 and ZnZr. Later it was experimentally proven that the superconductivity occurring in ZnZr is not volume but surface effect. Because the coexisting phase of UGe_2 is totally within the domain of ferromagnetic phase we have assumed that it is the presence of ferromagnetism that triggers the appearance of superconductivity under external pressure. The pressure participates only through the linear dependence of Curie temperature on P, namely $T_c = T_{c0}(1 - P/P_0)$, where T_{c0} is the Curie temperature at zero (ambient) pressure and P_0 is the pressure close to the critical P_c where ferromagnetism and superconductivity disappear. This free energy and the obtained results may be a good starting point for the description of (P, T) phase diagram in ferromagnetic superconductors.

The general form of Ginzburg–Landau free energy we use in our considerations of ferromagnetic superconductors with p-pairing is [12]:

$$\mathbf{F}[\vec{\psi}, \vec{M}] = \int d^3x \left[f_s(\vec{\psi}) + f_F(\vec{M}) + f_I(\vec{\psi}, \vec{M}) + \frac{\vec{B}^2}{8\pi} - \vec{B}.\vec{M} \right], \tag{1}$$

where the superconducting order parameter $\vec{\psi} = (\psi_1, \psi_2, \psi_3)$ is a 3D complex vector, $\vec{M}$ is the magnetization and both depend on the spatial variable $\vec{x}$. The magnetic induction is $\vec{B} = \vec{H} + 4\pi\vec{M} = \nabla \times \vec{A}$ with $\vec{H}$ – the external magnetic field and $\vec{A}$ – the magnetic vector potential.

The free energy density that describes the superconductivity in the absence of magnetization and external magnetic field $f_s(\vec{\psi})$ is expanded up to the fourth order in superconducting order parameter $\vec{\psi}$, including the respective anisotropic terms. Here we suppose tetragonal symmetry for superconductors with triplet Cooper pairing. Although all three uranium compounds UGe_2, URhGe and UCoGe have orthorhombic symmetry and the structure of superconducting order parameter for orthorhombic symmetry has been derived by general group considerations [13,14], we shall not consider for the time being the anisotropy in (x, y) plane, but only the uniaxial anisotropy, connected with the Ising-like anisotropy of magnetization:

$$\begin{aligned} \mathbf{f}_s(\vec{\psi}) = {} & f_{grad}(\vec{\psi}) + a_s|\vec{\psi}|^2 + \frac{b_s}{2}|\vec{\psi}|^4 + \frac{u_s}{2}|\vec{\psi}^2|^2 \\ & + \frac{v_s}{2}(|\overrightarrow{\psi_1}|^4 + |\overrightarrow{\psi_2}|^4). \end{aligned} \tag{2}$$

The above free energy density f_s of superconducting subsystem is written following the classification of superconducting states with triplet pairing deduced by general group symmetry approach in [15,16]. The obtained expression f_s possesses the symmetry of point group for tetragonal crystal symmetry, the symmetry of the spin rotations group, the time-reversal symmetry group and the gauge symmetry group by the way of its derivation.

The term $f_{grad}(\vec{\psi})$ gives the spatial dependence of the superconducting order parameter in the form:

$$\begin{aligned} \mathbf{f}_{grad}(\vec{\psi}) = {} & K_1(D_i\vec{\psi}_j)^*(D_iD_j) \\ & + K_2\left[(D_i\vec{\psi}_i)^*(D_j\vec{\psi}_j) + (D_i\vec{\psi}_j)^*(D_j\vec{\psi}_i)\right] \\ & + K_3(D_i\vec{\psi}_i)^*(D_i\vec{\psi}_i), \end{aligned} \tag{3}$$

where the symbol D_j stands for covariant differentiation $D_j = -i\hbar\partial/\partial x_i + 2|e|/cA_j$, and over the indices (i, j) summation is supposed, see [12]. The material parameters K_j are related to the effective mass tensor of anisotropic Cooper pairs [15,16].

The Landau material parameters in (2) are given by $a_s = \alpha_s[T - T_s(P)]$, where $T_s(P)$ is the critical temperature for pure superconducting system and $b_s > 0$. The parameters for Cooper-pair anisotropy u_s and crystal lattice anisotropy v_s within this approach are considered as undetermined material constants, characteristic of each particular substance and should be taken from the respective experimental data. All quantities α_s, b_s, u_s, v_s may be derived in principle from BCS-type microscopic models of superconductivity but vastly differ for weak and strong coupling limits.

The ferromagnetic energy density up to the forth order in magnetization $\vec{M}$ is denoted by f_F

$$\mathbf{f}_F = c_f \sum_{j=1}^{3} |\nabla_j M_j|^2 + a_f|\vec{M}|^2 + \frac{b_f}{2}|\vec{M}|^4. \tag{4}$$

Here $a_f = \alpha_f[T^n - T_f^n(P)]$ with a material parameter $\alpha_f > 0$, T is the temperature and T_f is the Curie temperature for ferromagnetic subsystem. The experimentally found ferromagnetic superconductivity in UGe_2, UCoGe and URhGe strongly depends on pressure and we take this into account by the dependence of Curie temperature $T_f(P)$ of pure ferromagnetic system on pressure P [11]. For $n = 1$ the usual Landau form of a_f is achieved; $n = 2$ describes the Stoner–Wohlfarth model [17]; $b_f > 0$. In the calculations below we will consider the case $n = 1$ and $n = 2$ will be only shortly discussed in connection with the phase diagram of UGe_2.

The interaction between the superconducting and magnetic order parameters is given by f_I, see [18,12]:

$$f_I = i\gamma_0\vec{M}.(\vec{\psi} \times \vec{\psi}^*) + \delta\vec{M}^2|\vec{\psi}|^2, \tag{5}$$

with $\gamma_0 \sim J$, where J is the ferromagnetic exchange constant.

Most experiments on UGe_2, UCoGe and URhGe (see the review paper [4]) show that the magnetization is of expressed Ising type. The strong uniaxial anisotropy of magnetic moment plays an important role for proposed magnetic mediated mechanisms for the appearance of triplet pairing favored by longitudinal magnetic fluctuations [7]. Recently it has been shown that in the fluctuation region UGe_2 and URhGe does not belong to the 3D Ising universality class [19]. But in this paper we will use the Landau approach so the critical fluctuations will be not considered and the critical exponents are the mean-field ones.

The uniaxial magnetic anisotropy means that magnetic moment in the above equations can be represented in the form $\vec{M} = (0, 0, M_z = M)$ by choosing z as the easy axis of magnetization. As we study only uniform phases in the absence of external magnetic field, $H = 0$, we will drop the dependence on the spatial variables of magnetization $\vec{M}$ and superconducting order parameter $\vec{\psi}$ and write for the free energy density $f_u = F/V$, where by f_u we have denoted the uniform part of free energy density Eq. (1) with V – the volume. Then the uniform magnetic and superconducting order parameters will depend only on T and P. To facilitate our considerations we make the uniform part of free energy f_u dimensionless with the help of the relation [11]:

$$f = \frac{f_u}{b_f M_0^4}, \tag{6}$$

where $M_0 = \alpha_f T_{f0}/\sqrt{b_f}$ is the magnetic moment at $T = 0$, $P = 0$ with T_{f0} the Curie temperature at zero pressure in the absence of