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Effects of the reservoir squeezing on the precision of parameter estimation

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The effects of reservoir squeezing on the precision of parameter estimation are investigated analytically based on non-perturbation procedures. The exact analytic quantum Fisher information (QFI) is obtained. It is shown that the QFI depends on the estimated parameter and its decay could be reduced by the squeezed reservoir compared with thermal (vacuum) reservoir, in particular, if the squeezing phase matching is satisfied.

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1. Introduction

The parameter estimation is one of most important ingredients in various fields of both the classical and quantum worlds [1,2]. The task of quantum estimation is not only to determine the value of unknown parameters but also to give the precision of the value. It is a vital issue on how to improve the estimation precision which is closely related to the quantum Cramér–Rao inequality and quantum Fisher information (QFI) [3–10] that determines the bound of the parameter's sensitivity theoretically by [11,12]

$$\delta(\phi) \geq 1/\sqrt{\nu \mathcal{F}_\phi}, \tag{1}$$

where ν means the time of experiments, and

$$\mathcal{F}_\phi = \mathrm{Tr}(\rho_\phi L_\phi^2) \tag{2}$$

is the QFI with the symmetric logarithmic derivative L_ϕ defined by $2\partial_\phi \rho_\phi = L_\phi \rho_\phi + \rho_\phi L_\phi$. Eq. (1) implies that the larger QFI means higher sensitivity of the parameter estimation.

The pioneer work on the quantum parameter estimation was proposed by Caves [13] who showed that the precision of phase estimation can beat the shot-noise limit (standard quantum limit). Later, lots of works with the similar aims are proposed, such as based on maximally correlated states [14], N00N states [15–17], squeezed states [18,19], or generalized phase-matching condition [20], and so on. In practical scenarios, it is inevitable for a quantum system to interact with environments, the precision of quantum estimation will be influenced by different extents [21–23]. In recent years, enormous effects have been devoted to how to improve the precision of parameter estimation in the case of open systems. For example, the precision spectroscopy using entangled state in the presence of Markovian dephasing [24] and non-Markovian noise [25] is investigated; the QFI under decoherence channels [26] or in a quantum-critical environment [27] is analyzed; the QFI measured experimentally with photons and atoms is reported [28,29]; it is also reported that the QFI subject to non-Markovian thermal environment [30] could show revival and retardation loss; the parameter-estimation precision in noisy systems could be enhanced by dynamical decoupling pulses [31], redesigned Ramsey-pulse sequence [32] or error correction [33] is also shown; noisy metrology beyond the standard quantum limits is possible when the noise is concentrated along some spatial direction [34]. However, if the environment we considered is a squeezed reservoir, how the QFI is influenced by the reservoir's parameters?

The squeezed reservoir has been widely studied in quantum information processing. For example, the squeezed light (reservoir) [35] or finite-bandwidth squeezing [36,37] for inhibition of the atomic phase decays and its application in microscopic Fabry–Pérot cavity [38]. In addition, some other considerations of the squeezing reservoir were also discussed, such as the quantum entanglement dynamics [39], heat engine recycle [40], geometric phase observable [41], etc. The physical realization of the squeezed reservoir has been proposed both in theory and in experiment based on various techniques such as the four-wave mixing [42], the parametric down conversion [43], the suitable feedback of the output signal corresponding to a quantum nondemolition measurement

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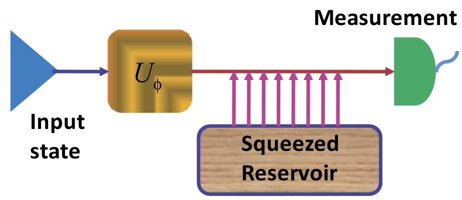

Fig. 1. (Color online.) The scheme of parameter estimation setup. After operated by a single qubit phase gate, the system will undergo a squeezed reservoir. The QFI can be obtained via optimal measurement.

of an observable [44], the control of the parameter of the driven laser [45], quantum conversion of squeezed vacuum states [46] or energy-level modulation [47], the atomic systems in cavity QED [48] or dissipative optomechanics system [49] and so on. The reduction of the radiative decay of the atomic coherence in squeezed vacuum has been realized in the superconducting circuit and microwave-frequency cavity system [50].

In this paper, we investigate the effects of reservoir squeezing on the QFI based on the non-perturbation processing [51]. We consider a phase estimation scheme in which a two-level system with an imposed unknown phase interacts with a squeezed reservoir before the final optimal measurements. To find the influences induced by the reservoir, we derive the non-perturbative master equation by the path integral method [51]. In terms of the master equation, we obtain the exact analytic expression of QFI which is related to the precision of parameter estimation. It can be found that the QFI depends on the estimated parameter and the decay of QFI can be reduced by the squeezed reservoir compared with thermal (vacuum) reservoir. In particular, if the appropriate squeezing phase matching condition is satisfied, the decay of QFI can be prevented prominently by the reservoir squeezing.

This paper is organized as follows. In Section 2, the parameter estimation scheme is introduced and the non-perturbation master equation is obtained. In Section 3, the exact analytic expression of QFI for the estimated parameter is obtained and the effects of reservoir squeezing on the QFI are investigated. The conclusions are given in the end.

2. Parameter estimation scheme

2.1. The scheme

The setup of the parameter estimation is sketched in Fig. 1. The input state is a two-level superposed state $|\psi_{in}\rangle = (|e\rangle + |g\rangle)/\sqrt{2}$. After the phase gate ($U_\phi = |g\rangle\langle g| + e^{i\phi}|e\rangle\langle e|$) is operated on the input state $|\psi_{in}\rangle$, the output state is given by

$$\rho_{out} = U_\phi |\psi_{in}\rangle\langle\psi_{in}| U_\phi^\dagger. \tag{3}$$

Let the system (ρ_{out}) interacts with a squeezed reservoir, the quantum Fisher information (QFI) of the final state can be obtained via optimal measurement. The inverse of square root of the QFI is related to the precision of parameter estimation according to Eq. (1) regardless of the experiment times ν.

2.2. The Hamiltonian

The initial state of the squeezed reservoir coupled to the system is given by

$$\rho_{bath} = \prod_k S_k(r,\theta)\rho_{th} S_k(r,\theta)^\dagger, \tag{4}$$

where the squeezed operator $S_k(r,\theta)$ and the thermal state ρ_{th} are given by

$$S_k(r,\theta) = \exp\left(\frac{1}{2}re^{-i\theta}b_k^2 - \frac{1}{2}re^{i\theta}b_k^{\dagger 2}\right), \tag{5}$$

$$\rho_{th} = \frac{\exp(-\beta\omega_k b_k^\dagger b_k)}{\mathrm{Tr}\exp(-\beta\omega_k b_k^\dagger b_k)}. \tag{6}$$

Here r is the squeezed parameter, θ is the reference phase of squeezed field and the parameter $\beta = 1/(kT)$ with k and T denoting the Boltzmann constant and temperature, respectively. Noting that the thermal state ρ_{th} will become the vacuum state $|0\rangle\langle 0|$ if the temperature $T = 0$, whilst the environment will become the squeezed vacuum reservoir $\prod_k S_k(r,\theta)|0\rangle\langle 0|S_k(r,\theta)^\dagger$ [52,53].

The total Hamiltonian of the system and reservoir reads

$$H = H_s + H_{bath} + H_{int}, \tag{7}$$

with

$$H_s = \omega_0\sigma_z/2, \tag{8}$$

$$H_{bath} = \sum_k \omega_k b_k^\dagger b_k, \tag{9}$$

$$H_{int} = \sum_k \left(g_k\sigma_+ b_k + g_k^*\sigma_- b_k^\dagger\right), \tag{10}$$

where ω_0 denotes the transition frequency of the two-level system, $\sigma_\pm$ is the raising/lowering operators of the system, $b_k^\dagger(b_k)$ is the creation (annihilation) operators of the squeezed reservoir and g_k is the strength of coupling between the system and environment.

2.3. The master equation of reduced density matrix

In order to get the master equation for the reduced system, we would like to employ the non-perturbative master equation which can be given, in the Schrödinger picture, by path integral method [51,54] as

$$\frac{\partial\rho_s}{\partial t} = -iL_s\rho_s - \int_0^t dt'\left\langle L_{int}e^{iL_0(t'-t)}L_{int}e^{-iL_0(t'-t)}\right\rangle_{bath}\rho_s, \tag{11}$$

where ρ_s denotes the reduced density matrix of the system, $\langle\cdot\rangle_{bath}$ denotes the partial trace of squeezed reservoir and L_0, L_s, L_{int} are the super operators defined by

$$L_s\rho = [H_s, \rho], \tag{12}$$

$$L_0\rho = [H_s + H_{bath}, \rho], \tag{13}$$

$$L_{int}\rho = [H_{int}, \rho]. \tag{14}$$

Assuming the initial state of system plus reservoir is product state $\rho_s(0)\otimes\rho_{bath}$, through tedious but straightforward derivation, the non-perturbative master equation in the interaction picture can be given by

$$\begin{aligned}\frac{\partial\rho_s}{\partial t} =& -(N+1)\alpha(t)\left(\sigma_+\sigma_-\rho_s - \sigma_-\rho_s\sigma_+\right)\\ &- (N+1)\alpha^*(t)\left(\rho_s\sigma_+\sigma_- - \sigma_-\rho_s\sigma_+\right)\\ &- N\alpha(t)\left(\rho_s\sigma_-\sigma_+ - \sigma_+\rho_s\sigma_-\right)\\ &- N\alpha^*(t)\left(\sigma_-\sigma_+\rho_s - \sigma_+\rho_s\sigma_-\right)\\ &+ 2\left(\alpha^*(t)M\sigma_+\rho_s\sigma_+ + \alpha(t)M^*\sigma_-\rho_s\sigma_-\right),\end{aligned} \tag{15}$$

where the coefficients N and M are represented by

$$N = n\left(\cosh^2 r + \sinh^2 r\right) + \sinh^2 r, \tag{16}$$

$$M = -\cosh r\sinh re^{i\theta}(2n+1) \tag{17}$$

with $n = 1/(\exp(\beta\omega) - 1)$ denoting average photon number.

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