Physics Letters A 379 (2015) 1283–1287

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Localized persistent spin currents in defect-free quasiperiodic rings with Aharonov–Casher effect

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ARTICLE INFO

Article history:
Received 5 December 2014
Received in revised form 9 February 2015
Accepted 16 February 2015
Available online 19 February 2015
Communicated by R. Wu

Keywords:
Persistent current
Spintronics
Quasiperiodic structure
Quantum interference

ABSTRACT

We propose strongly localized persistent spin current in one-dimensional defect-free quasiperiodic Thue–Morse rings with Aharonov–Casher effect. The results show that the characteristics of these localized persistent currents depend not only on the radius filling factor, but also on the strength of the spin–orbit interaction. The maximum persistent spin currents in systems always appear in the ring near the middle position of the system array whether or not the Thue–Morse rings array is symmetrical. The magnitude of the persistent currents is proportional to the sharpness of the resonance peak, which is dependent on the bandwidth of the allowed band in the band structure. The maximum persistent spin currents also increase exponentially as the generation order of the system increases.

1. Introduction

Over the past few decades, studies of spintronics devices [1–3] have attracted much attention for their theoretical and potential applications in quantum computing or spin filtering. For these devices, the discovery of a strongly localized mode is essential since it is related to the full width at half maximum (FWHM), which is an important parameter for electronic filters [4,5]. In terms of spintronic devices, one electronic structure that features one-dimensional ring with spin–orbit interaction (SOI), known as an Aharonov–Casher (AC) ring, is famous for its intriguing spin interference phenomena, such as a persistent current, which may possibly allow it to be applied in the qubit of quantum micro-circuits [6,7]. The other investigation of research subjects, such as spin-related modulation of the charge current [8], spin filters [4,9] and detectors [10], and so on have also been utilized intensively in ring geometries. Although, there have been many studies concerning persistent current on different arrangements of rings [11–17], most of these studies are mainly devoted to the observation and the magnification of a persistent current. Up to now, there are few studies about the characteristics of localized persistent currents.

Since the discovery of quasicrystals [18], many artificial and natural quasiperiodic materials have been proposed [19–28]. Both of their elementary physical phenomena and technological applications [21–23] have been attracted considerable attention. The Thue–Morse sequence is the typical quasiperiodic array that is used to study electronic characteristics [9,24,25]. For systems with quasiperiodic rings, most studies focus on the self-similarity and the scaling of the transmission spectra and the fracture energy spectra, based on spin-independent transport [26,27]. Little attention has been paid to the strongly localized modes of the persistent spin currents in Thue–Morse AC rings (TMARs). Therefore, it is of interest whether localized persistent currents can be produced by tuning the SOI strength, the generation order or other parameters of the TMARs and, if localized persistent current can be generated in TMARs, whether the magnitude and the location of the localized persistent current can be predicted. Such problems have not been discussed so far. In this paper, the effect of the SOI in units A and B on the occurrence of persistent current and resonances in the transmission spectra of the system is determined. We also further propose a periodic system for comparison with the results of the TMARs.

2. Model and formalism

The system studied in this paper is a one-dimensional quasiperiodic array structure that has two units, A and B, arranged in a Thue–Morse sequence. According to the iteration rule of Thue–Morse: $A \to AB$ and $B \to BA$, the structure for the lower generation order of the systems is given by $\nu \geq 2$, with $S_1 = \{A\}$, $S_2 = \{AB\}$, $S_3 = \{ABBA\}$ etc, as shown in Fig. 1(a). The number of rings, N_ν, in the νth order system is $N_\nu = 2^{\nu-1}$, with $N_2 = 2$ for $\nu = 2$. For comparison with the results of the TMARs, a periodic system that comprises binary periodic AC rings (BPARs) is also proposed,

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http://dx.doi.org/10.1016/j.physleta.2015.02.029
0375-9601/

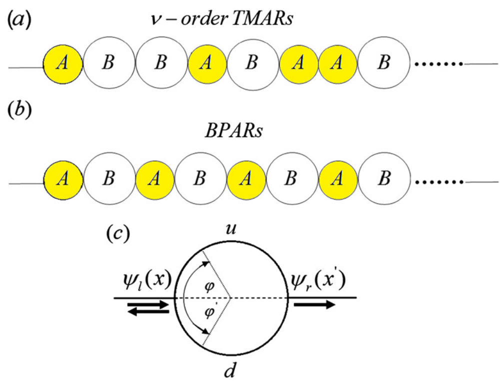

Fig. 1. (Color online.) A schematic diagram of a (a) νth order TMARs and a (b) BPARs system. (c) A schematic diagram of an ideal mesoscopic ring connected to two leads.

with $n_1 = \{AB\}$, $n_2 = \{ABAB\}$, $n_3 = \{ABABABAB\}$ etc., as shown in Fig. 1(b). Here, it is assumed that the width of the each element of the unit, arm or lead, is much less than the length of the one. The quantum energy levels for the transverse confinement are much greater than the ones for the longitudinal. Thus, we can simplify the problem to be one-dimensional for each element of the unit. In the presence of SOI, the Hamiltonian operator for a one-dimensional ring shown in Fig. 1(c) can be written as [29]

$$\widehat{H} = -\frac{\hbar^2}{2m^*a^2}\left(\frac{\partial^2}{\partial\varphi^2}\right) - i\frac{\alpha}{a}(\cos\varphi\sigma_x + \sin\varphi\sigma_y)\left(\frac{\partial}{\partial\varphi}\right) - i\frac{\alpha}{2a}(\cos\varphi\sigma_y - \sin\varphi\sigma_x) \tag{1}$$

where m^* is the effective mass of the carrier, a is the radius of the ring. The parameter, α, represents the SOI strength, which is measured in units of $\alpha_0 = \hbar^2/2m^*a = 0.6626 \times 10^{-11}$ eV m [30] and is controlled by a gate voltage with typical values in the range $(0.5\text{–}2.0) \times 10^{-11}$ eV m, for an InGaAs-based mesoscopic ring. We can solve the eigenvalue problem in a straightforward manner [31], and the energy eigenvalues are $E_n^\mu = (n - \Phi_{\mathrm{AC}}^\mu/2\pi)$, where $\mu = \pm$ and $\Phi_{\mathrm{AC}}^\mu = -\pi(1 - \mu\sqrt{1 + (\alpha/\alpha_0)^2})$ is the so-called AC phase. The unnormalized eigenstates have the general form, $\psi_{\mathrm{n}}^\mu = e^{in\varphi}\chi^\mu(\varphi)$, and the mutually orthogonal spinors, $\chi^\mu(\varphi)$, can be expressed in terms of the eigenvectors of the Pauli matrix, σ_z. The electronic wavefunction in the upper (u) and lower (d) arms can be written as

$$\psi_u^\mu(\varphi) = \left(a_+^\mu e^{in_+^\mu\varphi} + a_-^\mu e^{in_-^\mu\varphi}\right)\chi^\mu(\varphi), \tag{2a}$$

$$\psi_d^\mu(\varphi') = \left(b_+^\mu e^{-in_+^\mu\varphi'} + b_-^\mu e^{-in_-^\mu\varphi'}\right)\chi^\mu(-\varphi'). \tag{2b}$$

At a fixed energy, the dispersion relation yields the quantum numbers $n_\pm^\mu(E) = \pm\sqrt{E} + \Phi_{\mathrm{AC}}^\mu/2\pi$. The wave function in the left lead and right lead can be expanded as

$$\psi_l^\mu(x) = \left(e^{ikx} + e^{-ikx}r^\mu\right)\chi^\mu(0), \tag{2c}$$

$$\psi_r^\mu(x') = \left(e^{ikx'}t^\mu\right)\chi^\mu(\pi), \tag{2d}$$

where $\chi^+(\varphi) = (\cos(\theta/2)\ \ e^{i\varphi}\sin(\theta/2))^T$, $\chi^-(\varphi) = (\sin(\theta/2)\ \ -e^{i\varphi}\cos(\theta/2))^T$ and $k = \sqrt{2m^*E}/\hbar$ is the Fermi wave vector. By applying the continuity of the wave functions and the conservation of the spin current densities at the junctions of the leads and the ring [32], one finds

$$a_+^\mu e^{in_+^\mu\pi} + a_-^\mu e^{in_-^\mu\pi} = b_+^\mu e^{-in_+^\mu\pi} + b_-^\mu e^{-in_-^\mu\pi} = 1 + r^\mu, \tag{3a}$$

$$a_+^\mu + a_-^\mu = b_+^\mu + b_-^\mu = t^\mu, \tag{3b}$$

$$a_+^\mu e^{in_+^\mu\pi}\frac{n_+^\mu}{ka} + a_-^\mu e^{in_-^\mu\pi}\frac{n_-^\mu}{ka} - b_+^\mu e^{-in_+^\mu\pi}\frac{n_+^\mu}{ka} - b_-^\mu e^{-in_-^\mu\pi}\frac{n_-^\mu}{ka} = -1 + r^\mu, \tag{3c}$$

$$a_+^\mu e^{in_+^\mu\pi}\frac{n_+^\mu}{ka} + a_-^\mu e^{in_-^\mu\pi}\frac{n_-^\mu}{ka} - b_+^\mu e^{-in_+^\mu\pi}\frac{n_+^\mu}{ka} - b_-^\mu e^{-in_-^\mu\pi}\frac{n_-^\mu}{ka} = -t^\mu \tag{3d}$$

From Eqs. (3a)–(3d), the transmission and reflection amplitudes for spin polarization, $\mu = \pm$, for unit A in the presence of SOI, of strength α_A can be obtained

$$t_A^\mu = i\sin(ka_A\pi)\cos\left(\Phi_{\mathrm{AC}}^\mu/2\right)/D_A, \tag{4a}$$

$$r_A^\mu = \left[-3\sin^2(ka_A\pi) + 4\sin^2\left(\Phi_{\mathrm{AC}}^\mu/2\right)\right]/4D_A, \tag{4b}$$

where $D_A = \{\cos^2(\Phi_{\mathrm{AC}}^\mu/2) - [\cos(ka_A\pi) - i\sin(ka_A\pi)/2]^2\}$. Similarly, transmission and reflection amplitudes for unit B can be expressed in the same form, by replacing subscript A with B.

For different generation orders of the TMARs, S_ν, the transmission and reflection amplitudes are directly calculated, using the transfer matrix method [26,27]. The transfer matrix M_ν of a system with order ν is composed by the one with lower order $\nu - 1$ and its reverse, given by

$$M_\nu = \left\{M_{\nu-1}M_{\nu-1}^R\right\}\ (\nu \geq 2), \tag{5}$$

where $M_\nu^R = \{M_{\nu-1}^R M_{\nu-1}\}$, $M_1 = M_A$ and $M_1^R = M_B$. The transfer matrix for the J-connected TMARs can be expressed by

$$M_J = \begin{bmatrix} 1/t_J^{\mu^*} & -r_J^{\mu^*}/t_J^{\mu^*} \\ -r_J^\mu/t_J^{\mu^*} & 1/t_J^\mu \end{bmatrix}, \tag{6}$$

where t_J^μ and r_J^μ are respectively the total transmission and reflection amplitudes. Therefore, the total transmission probability for the J-connected TMARs is obtained by $T_J^\mu = |t_J^\mu|^2$. In the Landauer formalism, the conductance of the whole system array is given by

$$G = (e^2/h)\sum_{\mu=\pm}\left|t_J^\mu\right|^2. \tag{7}$$

3. Results and discussion

We start our discussion by considering the persistent currents in each ring of the two systems, which are TMARs and BPARs. The total spin current flow around a small energy interval is given by $I^\mu = (e/2\pi\hbar)T^\mu$, where T^μ is the total transmission coefficient. The current flow in the upper arm is expressed by $I_u^\mu = |a_+^\mu|^2 - |a_-^\mu|^2$, where a_+^μ and a_-^μ are the complex amplitudes of the forward and backward wavefunctions. When a net current flows through the arm from left (right) to right (left), the current is defined as positive (negative). The current in each arm is generally different from the others in the identical ring due to symmetry breaking via the unequal arm length, the AB effect, and the SOI, which leads to persistent currents [13]. According to the conservation of the current at the junction, $T^\mu = I_u^\mu - I_d^\mu$ for each ring, the persistent spin current, I_p^μ, in the ring is defined as [11,13]

$$I_p^\mu = \frac{(T^\mu - |I_u^\mu| - |I_d^\mu|)}{2}. \tag{8}$$

In this study, we examined the persistent spin currents of each ring in dimensionless units $I_p^\mu \to 2\pi\hbar I_p^\mu/e$.