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Stability of thermal convection of an Oldroyd-B fluid in a porous medium with Newtonian heating

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ABSTRACT

A linear stability analysis determining the onset of oscillatory convection of an Oldroyd-B fluid in a bounded two-dimensional rectangular porous medium generated by Newtonian heating is conducted. Influences of viscoelastic parameters and Biot number on the onset of oscillatory convection, preferred modes and patterns of disturbed temperature contours are discussed.

1. Introduction

Study of the stability of thermal convection in a fluid-saturated porous medium is very important for exploiting the dynamics of thermal convection and potential applications in many engineering fields, such as geothermal reservoirs, energy conservation, and cooling of electrical machines. In the past several decades, linear stability analysis of thermal convection in a Newtonian fluid-saturated porous medium has been conducted by many researchers based on the Darcy's law [1–5]. In contrast, publications on study of thermal convection in a viscoelastic fluid-saturated porous medium are far less than fruitful though interest in the flow and heat transfer characteristics for viscoelastic fluids increases rapidly in the past several years [6–9]. This is due to the lack of suitable models, as Darcy's law for Newtonian fluids, to describe the motion of viscoelastic fluids in porous media.

A linear stability analysis of a layer of pure viscoelastic fluid heated from below was first performed by Sokolov and Tanner [10] based on a simple constitutive relation for the viscoelastic fluid. They identified two possible instability mechanisms: exchange of stabilities and overstability. The former is similar to that for a Newtonian fluid, but the latter is unique to the viscoelastic fluid. Recently, Kim et al. [11] performed linear and nonlinear stability analyses of thermal convection for an Oldroyd-B fluid saturated in a horizontal porous layer with isothermal top and bottom boundaries based on a modified Darcy's law for description of the viscoelastic fluid motion in the porous layer. This model was first proposed by Alishaev and Mitzadajanzade [12], which is a macroscopic phenomenological model mimicking the Darcy's law but incorporating the viscoelastic effect. Subsequently, this model has been employed by several other researchers for studying thermal convection of viscoelastic fluids saturated in porous media [13–16]. Zhang et al. [17] investigated the stability of thermal convection in a porous cylinder saturated with a viscoelastic fluid; two different boundary heating conditions were considered: constant temperature heating and constant heat flux heating. They obtained the critical Rayleigh number for onset of thermal convection and the corresponding preferred mode of flow pattern for different combinations of the elastic parameters. Nevertheless, the heating conditions in these works are in the form of isothermal bottom boundary or uniform heat flux from below. No work has been conducted for the stability of a viscoelastic fluid in a porous medium subject to Newtonian heating, which is more commonly met in engineering applications.

Carslaw and Jaeger [18] were the first to introduce at the bottom wall a variable heat flux boundary condition expressed in terms of the Biot number, which represents a Robin boundary condition. This boundary condition is then referred to as Newtonian heating or forced convection heat transfer [19,20]. Kubitschek and Weidman [21] performed a linear stability analysis of a Newtonian fluid-saturated porous medium heated from below by Newtonian heating. They proved that when $Bi \to 0$, the thermal convection

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phenomenon approaches to that subject to uniform heat flux heating. On the other hand, when $Bi \to \infty$, it coincides with that under isothermal bottom heating. Therefore, thermal convection subject to different bottom heating conditions (i.e., Dirichlet, Neumann and Robin boundary conditions) can be achieved by tuning properly the value of the Biot number.

In this work, a two-dimensional linear stability analysis of an Oldroyd-B fluid saturated thin rectangular porous medium is performed under the Newtonian heating boundary condition. The critical Biot-modified Rayleigh numbers marking the onset of oscillatory convection are obtained numerically. The effects of the elastic parameters and the Biot number on the onset of oscillatory convection and preferred cellular modes are investigated, as well as the patterns of disturbed temperature distributions upon onset of oscillatory convection.

2. Model formulation

A bounded three-dimensional thin porous layer with height H^* and rectangular dimensions a^* and b^* is considered, in which b^* is much smaller than H^*. The four vertical boundaries of the model are adiabatic and impermeable. The impermeable horizontal top boundary is isothermal with a constant temperature T_1^*, while the impermeable horizontal bottom boundary is heated by Newtonian heating with external ambient temperature T_∞^* and convective heat transfer coefficient h. This porous medium has a permeability K and is saturated with an incompressible viscoelastic fluid which has a constant dynamic viscosity μ, a coefficient of thermal expansion β and a density ρ. The fluid-saturated porous medium has a thermal diffusivity κ. Here, flow of the Oldroyd-B fluid in the porous medium is described with the modified Darcy's law [12]. Therefore, the governing equations of the current problem, under the Oberbeck–Boussinesq approximation, are given as [11,13,14]

$$\nabla^* \cdot \mathbf{v}^* = 0 \tag{1}$$

$$\left(1 + \bar{\lambda}\frac{\partial}{\partial t^*}\right)\left(-\nabla^* p^* + \rho^* g\hat{\mathbf{k}}\right) = \frac{\mu}{K}\left(1 + \bar{\varepsilon}\frac{\partial}{\partial t^*}\right)\mathbf{v}^* \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + \left(\mathbf{v}^* \cdot \nabla^*\right)T^* = \kappa \nabla^{*2} T^* \tag{3}$$

$$\rho^* = \rho_1^*\left[1 - \beta\left(T^* - T_1^*\right)\right] \tag{4}$$

where $\mathbf{v}^*$ is the Darcy velocity with $\mathbf{v}^* = (u^*, v^*, w^*)$ in Cartesian coordinates, p^* the pressure, g the gravitational acceleration, $\bar{\varepsilon}$ and $\bar{\lambda}$ respectively the strain retardation time and the stress relaxation time, $\hat{\mathbf{k}}$ a unit vector along the z-direction which is vertically upward, and ρ_1^* the density at temperature T_1^*. The boundary conditions are given as

$$u^* = 0 \quad \text{at } x^* = 0, a^* \tag{5a}$$

$$v^* = 0 \quad \text{at } y^* = 0, b^* \tag{5b}$$

$$w^* = 0 \quad \text{at } z^* = -H^*, 0 \tag{5c}$$

$$\frac{\partial T^*}{\partial x^*} = 0 \quad \text{at } x^* = 0, a^* \tag{5d}$$

$$\frac{\partial T^*}{\partial y^*} = 0 \quad \text{at } y^* = 0, b^* \tag{5e}$$

$$T^* = T_1^* \quad \text{at } z^* = 0 \tag{5f}$$

$$h\left(T_\infty^* - T^*\right) = -k\frac{\partial T^*}{\partial z^*} \quad \text{at } z^* = -H \tag{5g}$$

In Eq. (5g), k is the effective thermal conductivity of the viscoelastic fluid-saturated porous layer.

3. Linear stability analysis

3.1. Linear stability equations

For convenience, a heat flux q can be introduced into the current work as [21]

$$q = \frac{Bi}{Bi+1} \cdot \frac{k\Delta T^*}{H^*} \tag{6}$$

Here, $\Delta T^* = T_\infty^* - T_1^* > 0$ and Bi is the Biot number defined as

$$Bi = \frac{hH^*}{k} \tag{7}$$

It should be noticed that q is not a constant heat flux in this problem, but a function of Bi and ΔT^*.

In order to perform the linear stability analysis, Eqs. (1)–(4) are first non-dimensionalized by scaling lengths with H^*, time with H^{*2}/κ, velocities with κ/H^*, pressure with $\kappa\mu/K$ and temperature with qH^*/k. Then θ and w are introduced to represent respectively the infinitesimal disturbances of the dimensionless temperature and vertical velocity over the pure conduction solution. As a consequence, Eqs. (1)–(4) can be, after eliminating the pressure term, rewritten as

$$\left(1 + \varepsilon\frac{\partial}{\partial t}\right)\nabla^2 w = Ra\left(1 + \lambda\frac{\partial}{\partial t}\right)\nabla_1^2\theta \tag{8}$$

$$\frac{\partial\theta}{\partial t} - w = \nabla^2\theta \tag{9}$$

where ε and λ are respectively the dimensionless relaxation and retardation characteristic times. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Ra in Eq. (8) is the Biot-modified Rayleigh number defined as

$$Ra = \frac{qK\rho_1\beta gH^{*2}}{\mu k\kappa} = \frac{Bi}{Bi+1} \cdot \frac{Kg\beta\Delta T^* H^*}{\nu\kappa} = \frac{Bi}{Bi+1} \cdot \overline{Ra} \tag{10}$$

In Eq. (10), ν is the kinematic viscosity of the viscoelastic fluid, $\overline{Ra}$ is the typical Rayleigh number in the case of a porous medium with constant temperature difference ΔT^* between isothermal bottom and top boundaries. The corresponding dimensionless boundary conditions of the disturbances are

$$w = 0 \quad \text{at } z = -1, 0 \tag{11a}$$

$$\frac{\partial\theta}{\partial x} = 0 \quad \text{at } x = 0, a \tag{11b}$$

$$\frac{\partial\theta}{\partial y} = 0 \quad \text{at } y = 0, b \tag{11c}$$

$$\theta = 0 \quad \text{at } z = 0 \tag{11d}$$

$$Bi\,\theta - \theta_z = 0 \quad \text{at } z = -1 \tag{11e}$$

where $a = \frac{a^*}{H^*}$ and $b = \frac{b^*}{H^*}$ are dimensionless side lengths. In the case when b^* is much smaller than H^*, i.e., $b \ll 1$, the problem can be simplified into a two-dimensional one.

3.2. The characteristic equation and solution procedure

Under the normal mode analysis, temperature and velocity disturbances are supposed to be horizontally periodic. Taking the adiabatic vertical boundary conditions into account, the solution should be of the form:

$$\theta = \cos(\alpha x)\cos(\beta y)\Theta(z)e^{\sigma t}, \quad \alpha = \frac{m\pi}{a}, \ \beta = \frac{n\pi}{b} \tag{12}$$

$$w = \frac{\partial\theta}{\partial t} - \nabla^2\theta = \left(\sigma + L^2 - D^2\right)\theta, \quad L^2 = \alpha^2 + \beta^2 \tag{13}$$