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Electromagnetic surface waves on a conducting cylinder

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ABSTRACT

We study propagation of electromagnetic surface waves on a metal–air interface in the case when the wave frequency is below the plasma frequency. We derive a reduced wave equation for a metal cylinder with a given radius of curvature. Using the Leontovich boundary condition we find solutions to this equation which we classify as outgoing and incoming surface waves. We derive the dispersion relations of the surface waves of both types and argue that the earlier studies overlooked the waves of the second type although they are the only type which can propagate on a planar metal–air boundary.

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1. Introduction

Recent developments that lead to extending of plasmonics to the mid- and far-infrared (terahertz) ranges of frequencies constitute an important direction of modern photonics [1–3]. Surface Plasmon Polaritons (SPPs) are partially longitudinal p-polarized evanescent electromagnetic waves that propagate along a metal–air interface as a result of collective electron oscillations coupled to an external electromagnetic field [4–7,2,8–10]. In practical applications it is important to understand how SPPs propagate along a curved surface [11,12,3]. For this reason, the theory of SPPs on a cylinder attracted a lot of attention in the past and recent years [13–20]. A dispersion equation for the transverse magnetic (TM) modes was derived by M. Berry in an artificial case of lossless metal [13] and later by K. Hasegawa et al. in a general case [15,16]. A similar equation appears in the theory of diffraction of a wave on a transparent cylinder [21–24]. It involves the Bessel and Hankel functions of a large argument and a large complex order which makes its numerical evaluation for practical problems difficult. Different approaches have been developed to overcome this difficulty as reviewed in Ref. [18] but none of them was especially successful.

The standard approach to the derivation of SPPs [13,15,16] assumes that the radial distribution of the magnetic field of a plasmon near the surface of a metallic cylinder is described by the Hankel function of the first kind, $\mathrm{H}_n^{(1)}(k_0 r)$, with $k_0 = \omega/c$, ω the frequency of the mode, and r the radius in cylindrical system of coordinates. The Hankel function of the second kind, $\mathrm{H}_n^{(2)}(k_0 r)$, is explicitly or implicitly discarded.

This approach is usually justified by the fact that at large distances, $k_0 r \gg 1$, the asymptotic representation of the Hankel function $\mathrm{H}_n^{(1)}$ describes a radial distribution of a wave propagating away from the cylinder, whereas $\mathrm{H}_n^{(2)}(k_0 r)$ stands for a wave, propagating to the cylinder. The situations however becomes more complicated if one takes into account that, in the case of finite conductivity of the metal, the index n, which has to be found from a dispersion equation, is a large complex number. In this case, $\mathrm{H}_n^{(1)}$ can experience a large exponential growth in the radial direction in the vicinity of the metal surface, meaning that the mode is not actually localized near the surface.

In our opinion, an adequate approach to the study of SPPs is to analyze their spatial (radial plus azimuthal) distribution in the vicinity of the metal surface, $r - R \ll R$, where R is the cylinder radius (see Fig. 1 below). We introduce a small parameter $\delta = \sqrt{a/R}$, where $a \sim r - R$ is a transverse localization length of the mode and derive equations in which small terms in parameter δ are systematically neglected. A similar method was previously successfully used in the theory of synchrotron radiation in a curved waveguide [25,26]. Here we use it for investigation of the Surface electromagnetic Waves (SW) on a metal–air interface with given and fixed radius of curvature. Solving these equations we obtain expressions for the fields in terms of the Airy–Fock functions, avoiding in this

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way the numerical difficulties arising when one deals with the Hankel functions of large argument and large order.

In this our study we find that, indeed, the localized solutions in the form of $H_n^{(1)}$ used in [13,15,16] and some subsequent papers (see e.g. [17,18]) may exist only in a narrow range of values of the dimensionless surface impedance ξ. On the contrary, we find that SWs which are expressed through the Hankel functions of the second kind $H_n^{(2)}(k_0 r)$, do exist for any ξ provided that the radius of the cylinder is not too small. We named these two types of surface waves correspondingly the outgoing and incoming surface waves. We also note that in a planar geometry an ordinary surface wave belongs to the type of incoming surface waves. As to the outgoing modes discussed in Refs. [15–17,19,18], they can be identified as creeping waves (CW). In contrast to the surface waves, the amplitude of CWs grows in radial direction in the vicinity of the metal; in Ref. [20] these waves were characterized as wave-guide modes.

The term "creeping waves" is used in the theory of diffraction [27,21,28–32] for the refracted waves over convex conducting or dielectric surfaces where an extra attenuation due to the curvature is present. The foundations of the theory were laid out in earlier works of V.A. Fock [33,34] who considered propagation of radio waves around the Earth. As explained in Ref. [27, Chapt. 5], the creeping waves rapidly damp in the shadow region created by a convex obstacle on the way of a wave propagation. They damp both in the direction of propagation and towards to the surface of the obstacle. In other words, the amplitude of a creeping wave grows with the distance along the normal to the surface from the shadow region to the lit one. On the contrary, the amplitude of the surface waves decays with the distance from the surface. This qualitative difference of SWs from CWs, which are also known as "Whatson modes", is discussed in Ref. [30].

The paper is organized as follows. A paraxial equation for electromagnetic waves near a curved surface is derived in Section 2. It is used in Section 3 to obtain dispersion equations of various surface electromagnetic waves on a curved surface; we introduce here the notion of outgoing and incoming surface waves. A numerical solution to these equations is discussed in Section 4. Analytical results are outlined in Section 5; for details of calculations we refer the reader to archived version of this paper [35]. All these sections deal with a convex geometry, where the metallic medium is a cylinder surrounded by air. A concave case, where a hollow conducting cylinder is filled by air, is briefly discussed at the end of Section 5. In Section 6 we show how the Airy–Fock functions are matched to the Hankel functions in a relevant range of parameters thus providing a bridge between our results and previous studies in Refs. [15–19]. Finally, in Section 7 we summarize main results.

2. Paraxial equation

In this section we will derive equations for the electric and magnetic fields near the surface of a conducting cylinder. The geometry of the problem and the choice of the coordinate system are shown in Fig. 1. We use the cylindrical coordinate system r, θ, z and the notation x for the difference $x = r - R$. The metal surface is located at $x = 0$.

We first assume that the metal occupies the region $x < 0$; the region $x > 0$ has the dielectric permeability $\varepsilon = 1$. This is so-called *convex case* [17], where a metallic circular cylinder is surrounded by an empty space (or by air). Later on, we will extend our results to the *concave case* by assigning an opposite sign to the radius R. As to the current and subsequent sections, except for Section 6, we suppose that $R > 0$.

We assume that all components of the field have the following dependence on time and the azimuthal angle θ

$$\boldsymbol{E}, \boldsymbol{H} \propto \boldsymbol{U}(r) \mathrm{e}^{-i\omega t + ikR\theta} \tag{1}$$

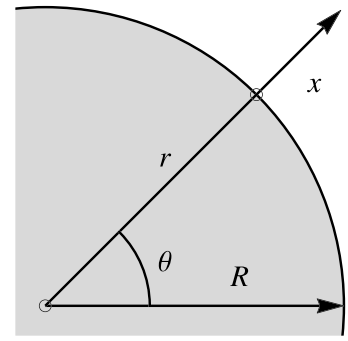

Fig. 1. Coordinate system with R the cylinder radius. A conducting medium occupies either an interior (convex geometry) or exterior (concave geometry) of the cylinder. The radial coordinate r is measured from the axis of the cylinder, and $x = r - R$ in counted from the cylinder surface. A transverse magnetic (TM) wave under consideration has magnetic field H_z directed along the axis z of the cylinder.

with amplitude $\boldsymbol{U}(r)$ independent of the coordinate z. Moreover, we restrict ourselves to the case of transverse magnetic (TM) waves although it has been recently reported [36] that transverse electric (TE) surface waves may exist on a cylindrical metal–air interface in contrast to the absence of such waves in a planar geometry (see e.g. [1,2,6,37,38]). Hence, we assume that the magnetic field

$$\boldsymbol{H} = \{0, 0, H_z\}$$

has only the z-component, and the electric field

$$\boldsymbol{E} = \{E_r, E_\theta, 0\}$$

has both the r- and θ-components.

To keep track of order of magnitudes, we will assume that $x \sim a$ is of the order of δ^0 and the radius R is of order of δ^{-2}, where $\delta = (a/R)^{1/2} \ll 1$ is a formal small parameter of the problem. The wavenumber k and the frequency $\omega \approx kc$ are assigned the order of δ^{-1}. We will clarify our ordering scheme in Section 3 in more details.

From the radial component

$$-\frac{i\omega}{c} E_r = \frac{ikR}{r} H_z$$

of the Maxwell equation $\operatorname{rot} \boldsymbol{H} = (1/c)\, \partial \boldsymbol{E}/\partial t$ (here and below we use the Gaussian system of units), we find to the lowest order

$$E_r = -H_z. \tag{2}$$

This is the relation that holds in a plane electromagnetic wave; in our case it is satisfied approximately to order δ^2. From the azimuthal component of the same equation

$$-i\,\frac{\omega}{c} E_\theta = -\frac{\partial H_z}{\partial r},$$

we express E_θ in terms of H_z,

$$E_\theta = -\frac{ic}{\omega} \frac{\partial H_z}{\partial x}. \tag{3}$$

As to the function $H_z(r)$, it obeys the equation

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial H_z}{\partial r} + \left(\frac{\omega^2}{c^2} - \frac{k^2 R^2}{r^2} \right) H_z = 0. \tag{4}$$

Substituting $r = R + x$ and expanding in small ratio x/R we find

$$\frac{\partial^2 H_z}{\partial x^2} + 2 \frac{\omega^2}{c^2} \left(\frac{x}{R} - \Lambda \right) H_z = 0, \tag{5}$$

where

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