Physics Letters A 372 (2008) 5887–5890

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

# Vehicular motion through a sequence of traffic lights controlled by logistic map

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ARTICLE INFO

*Article history:*
Received 2 May 2008
Received in revised form 22 July 2008
Accepted 28 July 2008
Available online 3 August 2008
Communicated by A.R. Bishop

*PACS:*
89.40.+k
05.45.-a
82.40.Bj

*Keywords:*
Traffic dynamics
Signal control
Logistic map
Nonlinear map
Complex system

ABSTRACT

We study the dynamical behavior of a single vehicle through the sequence of traffic lights controlled by the logistic map. The phase shift of traffic lights is determined by the logistic map and varies from signal to signal. The nonlinear dynamic model of the vehicular motion is presented by the nonlinear map including the logistic map. The vehicle exhibits the very complex behavior with varying both cycle time and logistic-map parameter $a$. For $a > 3$, the dependence of arrival time on the cycle time becomes smoother and smoother with increasing $a$. The dependence of vehicular motion on parameter $a$ is clarified.



## 1. Introduction

Physics, other sciences and technologies meet at the frontier area of interdisciplinary research. Recently, transportation problems have attracted much attention in the fields of physics [1–5]. The concepts and techniques of physics are being applied to such complex systems as transportation systems. The traffic flow, pedestrian flow, and bus-route problem have been studied from a point of view of statistical mechanics and nonlinear dynamics [6–15]. The interesting dynamical behaviors have been found in the transportation system. The jams, chaos, and pattern formation are typical signatures of the complex behavior of transportation.

Mobility is nowadays one of the most significant ingredients of a modern society. In urban traffic, vehicles are controlled by traffic lights to give priority for a road because they encounter at crossings. Brockfeld et al. have studied optimizing traffic lights for city traffic by using a CA traffic model [16]. They have clarified the effect of signal control strategy on vehicular traffic. Also, they have shown that the city traffic controlled by traffic lights can be reduced to a simpler problem of a single-lane highway. Sasaki and Nagatani have investigated the traffic flow controlled by traffic lights on a single-lane roadway by using the optimal velocity model [17]. They have derived the relationship between the road capacity and jamming transition.

In real traffic, the vehicular traffic depends highly on the control of traffic lights. Until now, one has studied the periodic traffic controlled by a few traffic lights. It has been concluded that the periodic traffic does not depend on the number of traffic lights [16,17]. Very recently, a few works have been done for the traffic of vehicles moving through an infinite series of traffic lights with the same interval. The effect of cycle time on vehicular traffic has been clarified [18–22].

Very recently, Lammer and Helbing have studied the effect of self-controlled signals on vehicular flow based on fluid-dynamic and many-particle simulations [23].

Generally, the traffic lights are controlled by either synchronized or delayed strategies. In the synchronized strategy, all the signals change simultaneously and periodically where the phase shift has the same value for all signals. In the delayed strategy, the signal changes with a certain time delay between the signal phases of two successive intersections. The change of traffic lights propagates backward like a green wave. The delayed strategy is called the green-wave strategy. Thus, the vehicular traffic depends highly on the signal's strategy. The operator will be able to control the traffic signal by the use of the other strategy. Specifically, one can manage the phase shifts of signals.

In this Letter, we apply the logistic map to the signal's strategy [24,25]. We control the traffic light by using the logistic map.

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0375-9601/$ – see front matter 
doi:10.1016/j.physleta.2008.07.063

The phase shifts of signals are given by the logistic map. The phase shift of traffic lights changes from signal to signal. We present the nonlinear dynamic model for the vehicular motion through the sequence of traffic lights. We investigate the dynamical behavior of a single vehicle. We clarify the dynamical behavior of a single vehicle through the sequence of signals by varying both cycle time and logistic-map parameter.

## 2. Nonlinear dynamic model

We consider the motion of a single vehicle going through the infinite series of traffic lights. The traffic lights are numbered, from upstream to downstream, by $1, 2, 3, \ldots, n, n+1, \ldots$. The traffic lights are positioned with the same interval on a roadway where the interval between signals $n-1$ and $n$ is indicated by $l$. All the signal changes periodically with period $t_s$. Period $t_s$ is called as the cycle time. The phase shift of signals varies from signal to signal. The vehicle moves with the mean speed $v$ between a traffic light and its next light.

In the synchronized strategy, all the traffic lights change simultaneously from red (green) to green (red) with a fixed time period $(1-s_p)t_s$ $(s_p t_s)$. The period of green is $s_p t_s$ and the period of red is $(1-s_p)t_s$. Fraction $s_p$ represents the split which indicates the ratio of green time to cycle time.

The signal timing is controlled by offset time $t_{\text{offset}}$. The offset time means the difference of phase shifts between two successive signals. In the delayed (green wave) strategy, the phase shift of signal $n$ is given by $t_{\text{phase}}(n) = n t_{\text{offset}}$ where the phase shift is indicated by $t_{\text{phase}}(n)$. Then, the signal switches from red to green in green wave way. Here, we control the phase shift of signals with the use of the logistic map. The phase shift varies with signal $n$. When logistic-map parameter $a$ is higher than critical value $a_c = 3.56$, the phase shift changes irregularly from signal to signal.

When a vehicle arrives at a traffic light and the traffic light is red, the vehicle stops at the position of the traffic light. Then, when the traffic light changes from red to green, the vehicle goes ahead. On the other hand, when a vehicle arrives at a traffic light and the traffic light is green, the vehicle does not stop and goes ahead without changing speed.

We define the arrival time of the vehicle at traffic light $n$ as $t(n)$. The arrival time at traffic light $n+1$ is given by

$$t(n+1) = t(n) + \frac{l}{v} + \big(r(n) - t(n)\big) H\left(t(n) + t_{\text{phase}}(n) - \left[\operatorname{int}\left(\frac{t(n) + t_{\text{phase}}(n)}{t_s}\right) t_s\right] - s_p t_s\right)$$

$$\text{with } r(n) = \left(\operatorname{int}\left(\frac{t(n) + t_{\text{phase}}(n)}{t_s}\right) + 1\right) t_s - t_{\text{phase}}(n), \tag{1}$$

where $H(t)$ is the Heaviside function: $H(t) = 1$ for $t \geqslant 0$ and $H(t) = 0$ for $t < 0$. $H(t) = 1$ if the traffic light is red, while $H(t) = 0$ if the traffic light is green. $l/v$ is the time it takes for the vehicle to move between lights $n$ and $n+1$. $r(n)$ is such time that the traffic light just changed from red to green. The third term on the right-hand side of Eq. (1) represents such time that the vehicle stops if traffic light $n$ is red. The number $n$ of iteration increases one by one when the vehicle moves through the traffic light. The iteration corresponds to the going ahead on the highway.

By dividing time by the characteristic time $l/v$, one obtains the nonlinear equation of dimensionless arrival time:

$$T(n+1) = T(n) + 1 + \big(R(n) - T(n)\big) H\left(T(n) + T_{\text{phase}}(n) - \left[\operatorname{int}\left(\frac{T(n) + T_{\text{phase}}(n)}{T_s}\right) T_s\right] - s_p T_s\right)$$

$$\text{with } R(n) = \left(\operatorname{int}\left(\frac{T(n) + T_{\text{phase}}(n)}{T_s}\right) + 1\right) T_s - T_{\text{phase}}(n), \tag{2}$$

where $T(n) = t(n)v/l$, $R(n) = r(n)v/l$, $T_{\text{phase}}(n) = t_{\text{phase}}(n)v/l$ and $T_s = t_s v/l$. Phase shift $T_{\text{phase}}(n)$ is given by the logistic map

$$T_{\text{phase}}(n+1) = a T_{\text{phase}}(n)\big(1 - T_{\text{phase}}(n)\big). \tag{3}$$

Thus, the dynamics of the vehicle is described by the nonlinear map (2) with logistic map (3).

It will be expected that the vehicular motion exhibits a complex behavior. We study how the vehicular motion changes by varying both cycle time $T_s$ and logistic-map parameter $a$.

## 3. Simulation result

We investigate the effect of phase shift on the motion of a single vehicle through the series of traffic lights by iterating nonlinear map (2) with logistic map (3). We calculate the arrival time at traffic light $n$ when the vehicle goes ahead through the series of traffic lights on roadway. We study how the arrival time varies with cycle time for various values of logistic-map parameter $a$. We clarify the dynamical behavior of the signal traffic controlled by the logistic map. We compare the vehicular traffic with that controlled by the synchronized strategy. We restrict ourselves to the case of $s_p = 0.5$.

Fig. 1 shows the plots of rescaled arrival times $T(1000) - 1000$ and $(T(2000) - 2000)/2$ against cycle time $T_s$ at signals $n = 1000$ and $n = 2000$ far from the origin. Diagrams (a) and (b) indicate, re-

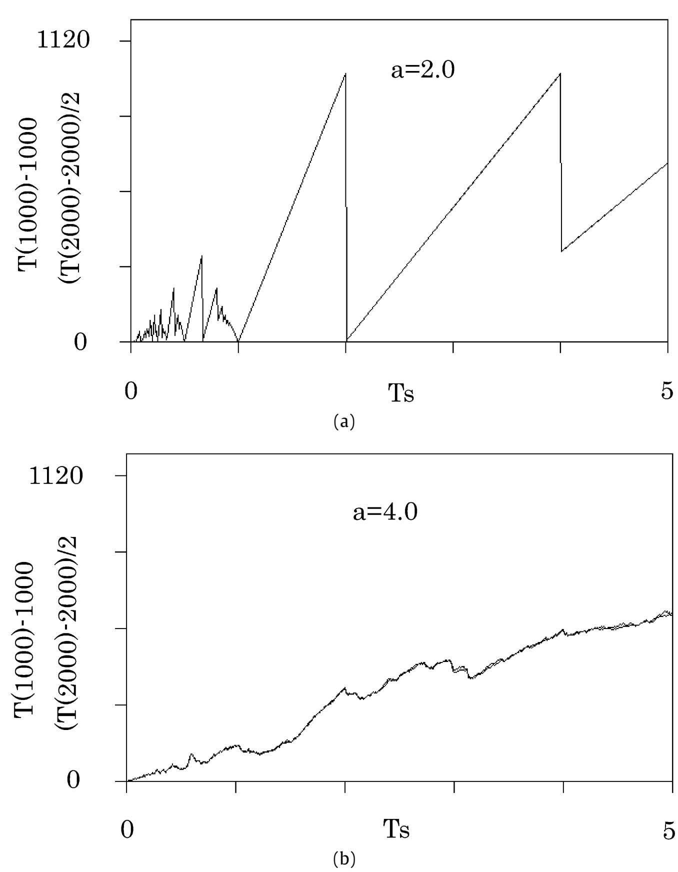


**Fig. 1.** Plots of rescaled arrival times $T(1000) - 1000$ and $(T(2000) - 2000)/2$ against cycle time $T_s$ at signals $n = 1000$ and $n = 2000$ far from the origin. Diagrams (a) and (b) indicate, respectively, the rescaled arrival times for $a = 2.0$ and $a = 4.0$ where $a$ is the logistic-map parameter.