Physics Letters A 379 (2015) 767–773

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Spatial prisoner's dilemma games with increasing neighborhood size and individual diversity on two interdependent lattices

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ARTICLE INFO

Article history:
Received 14 November 2014
Received in revised form 29 December 2014
Accepted 30 December 2014
Available online 6 January 2015
Communicated by C.R. Doering

Keywords:
Cooperation enhancement
Prisoner's dilemma game
Increasing neighborhood size
Individual diversity
Interdependency

ABSTRACT

We present an improved spatial prisoner's dilemma game model which simultaneously considers the individual diversity and increasing neighborhood size on two interdependent lattices. By dividing the players into influential and non-influential ones, we can discuss the impact of individual diversity on the cooperative behaviors. Meanwhile, we implement the utility interdependency by integrating the payoff correlations between two lattices. Extensive simulations indicate that the optimal density of influential players exists for the cooperation to be promoted, and can be further facilitated through the utility coupling. Current results are beneficial to understanding the origin of cooperation among selfish agents among realistic scenarios.

1. Introduction

Understanding the persistence and emergence of cooperative behaviors between unrelated or selfish agents, ranging from cellular organisms and their organs in biology to complex human individuals in society, poses a great challenge among scientific communities [1] and becomes an interdisciplinary topic which attracts intensive interests of mathematicians, biologists, physicists, social scientists *etc.* [2]. A powerful framework that has given rise to much deeper insights into this issue is provided by the evolutionary game theory [3], which allows the researchers to quantitatively formulate the most significant social interactions in real-world systems, including the social welfare, price decision, strategy conflict or dilemma and so on. The paradox between collective and individual rationality renders the fact that it is much more difficult to shed light on the widespread coordination or cooperation behaviors [4]. Several important mechanisms, such as kin selection [5], direct or indirect reciprocity [6–8], group interaction [9], spatial reciprocity [10], networking reciprocity [11–13] and so on, have been identified as effective means to support the evolution of cooperation. Among them, network science [14,15] offers a brand new tool to help us to explore the evolutionary game theory on complex structured populations, which represents one of the most intriguing dynamical processes on top of complex networks [16,17]. In reality, analyzing the structure of and various dynamics on networks has become an active topic in the recent years [18–23].

With the continuing development of network science, the structure or evolving pattern of real-world systems has often been modeled as the interdependency and multiplexing of two or more submodules [24,25], on which the structural properties and dynamical behaviors exhibit some distinct phenomena from single, isolated networks [26]. Thus, interdependent or multiplexing network becomes a novel platform for us to account for the emergency of cooperation through the network reciprocity [27]. For example, Wang et al. investigated the evolution of cooperation on two interdependent networks that are correlated each other by means of a utility function to measure an individual fitness, which combines the payoff of focal player on one network and payoffs of corresponding partner or/and its neighbors on the other network [28–30], and they find that this new kind of fitness evaluation can effectively promote the level of cooperation. Gómez-Gardeñes et al. discussed the issue of emergency of prisoner's dilemma cooperation using multiplexing populations in which each individual participates in the game playing on several layers at the same time [31], and they find that the resilience of cooperation for extremely large temptation parameter is elevated by the multi-layer structure. Santos et al. explored the evolutionary dynamics adopting a kind of bi-

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http://dx.doi.org/10.1016/j.physleta.2014.12.051
0375-9601/

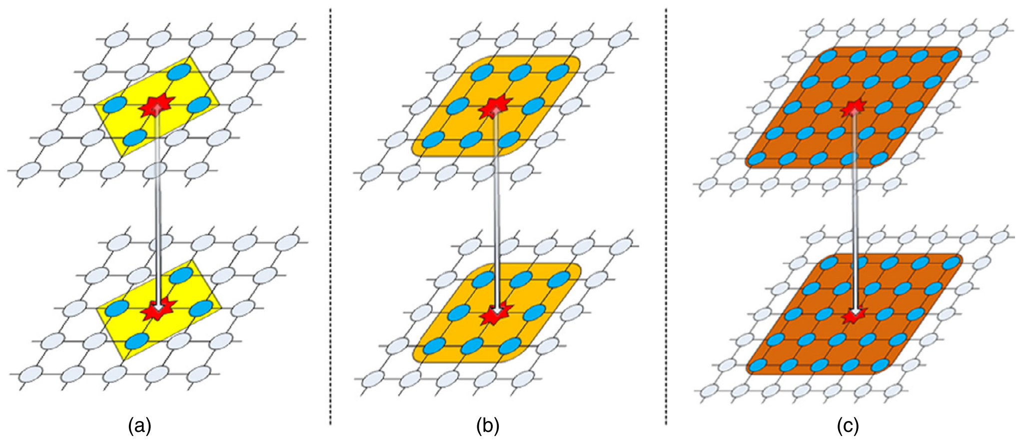

Fig. 1. Three typical neighborhood setups, and each focal player can only play the PDG with nearest neighbors included within shaded areas. Although the individual utility can be related into the corresponding partner on the opposite lattice, the strategy spreading can be allowed in the same lattice. From panel (a) to (c), the neighborhood size k is set to be 4, 8 and 24, respectively.

ased strategy imitation on two different networks in which the prisoner's dilemma game (PDG) is played on one network while the snowdrift game (SDG) is played on the other one, and the results indicate that the biased imitation is beneficial to the PDG but detrimental to the SDG [32]. Jiang and Perc studied the cooperation behavior between groups with regular, random and scale-free topology, and reveal that too many or few between-group links do not facilitate the spreading of cooperative strategy between group with different topology [33]. Xia et al. proposed an improved traveler's dilemma game model on two coupled lattices to talk about the impact of coupling effect during the strategy imitation on the cooperation, and it is clearly shown that the cooperation can be promoted only if the model parameter R surpasses a specific threshold [34]. Wang et al. introduced the co-evolution between strategy and network interdependence to investigate whether it can lead to the elevated level of cooperation in the PDG [35]. In addition, Szolnoki and Perc also showed that sharing information about strategy choice between players locating on two different networks reinforces the evolution of cooperation [36]. The above-mentioned works have demonstrated that the interdependency or multiplexing radically changes the evolutionary behaviors [37–39] when compared to those taking place upon single, isolated populations.

Nevertheless, any player on each network is usually assumed to be an equivalent and independent agent, which owns the identical payoff evaluation and strategy imitation process. In reality, except for the structural in-homogeneity, individuals may often exhibit, to a greater extent, the behavioral heterogeneity or diversity, such as personal learning capability, immunity, aspiration level and so on [40,41]. For example, Szabó and Szolnoki discussed the role of individual diversity characterized by two types of players with different strategy adoption probability in the spatial prisoner's dilemma game (PDG) model [42], and found that this kind of in-homogeneity in strategy transfer can yield the promotion of cooperation within a moderate density of influential players. Additionally, Zhu et al. integrated the heterogeneity in strategy transfer into the public goods game (PGG), and observed that the collective cooperation can be greatly enhanced under the intermediate portion of influential players [43]. Meanwhile, Perc et al. present a PDG model to illustrate the impact of density and interconnectedness of influential players on social welfare [44]. Therefore, in this Letter, based on the spatial PDG model, we further investigate the evolution of cooperation on interdependent lattices where the individual neighborhood setup is different from the traditional von-Neumann neighborhood, and simultaneously the players can hold two types of distinct strategy imitation probabilities characterizing the individual behavior heterogeneity and diversity. A large plethora of simulations validate the fact that the individual diversity and coupling of utility evaluation can greatly promote the evolution of cooperation.

The remainder of this Letter is structured as follows. In Section 2, we firstly introduce the prisoner's dilemma game model on interdependent lattices in detail. Then, extensive numerical simulation results and discussions are presented in Section 3. Finally, the concluding remarks are presented in Section 4.

2. Model

In our model, the whole system is composed of two $L \times L$ lattices on which $2 * N$ ($N = L^2$) players are located, that is, each site on these two lattices rightly holds a player. Initially, each player x can randomly select one of two pure strategies: Cooperation ($s_x = C$) or Defection ($s_x = D$). Then, each player will combat the game with his/her nearest neighbors to collect his/her total payoff at the current game round. Here, the game model we choose is the typical PDG which represents the strictest dilemma a player faces in the real-world life. This game can be briefly introduced as follow. The players will obtain the reward (R) or punishment (P) payoff if they adopt the same strategy; Conversely, the defective player owns the highest payoff (T, temptation to defect) and the cooperative one gets the sucker's payoff (S) if they apply the different strategies. In the PDG, the constraints $T > R > P > S$ and $2R > S + T$ often need to be satisfied so as to denote the dilemma an individual confronts. Each player's strategy ($s_x = C$ or $s_x = D$) can also be expressed with the following unit vector,

$$s_x = C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad s_x = D = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

Making use of this unit vector, we can easily utilize a simple matrix algebra to calculate the total income P_x of player x, which is accumulated from the payoff obtained by playing the PDG with all his/her nearest neighbors (say, y), and P_x can be computed as follows,

$$P_x = \sum_{y \in \Omega_x} s_x^+ A s_y \tag{2}$$

where Ω_x represents the set of all nearest neighbors of player x and k denotes the size of neighborhood ($k = |\Omega_x|$), s_x^+ is the transpose of the strategy vector s_x, s_y is the strategy vector of the nearest neighbor y and A denotes the payoff matrix of PDG. Here, we consider three different neighborhood setup illustrated in