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Generation of grid multi-scroll chaotic attractors via switching piecewise linear controller

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A B S T R A C T

In this Letter, a novel method is developed for generating grid multi-scroll chaotic attractors using switching piecewise linear controller. First, a third-order linear system is designed to ensure that its unique equilibrium point belongs to a saddle-focus type with index 2 and the corresponding eigenvalues satisfy Shilnikov conditions. Then, by three different types of switching control strategies, the equilibrium point can be extended along both xy plane and z axis direction, so as to generate grid multi-scroll chaotic attractors. The dynamical behaviors are further analyzed. Moreover, an improved module-based circuit is designed for realizing 5×3 and 4×4 grid scroll chaotic attractors, and the experimental results are also obtained, which is consistent with the numerical simulations.

1. Introduction

In recent years, one could find the diverse studies and applications of chaos in nonlinear system, information science as well as in engineering applications. Since the proposal of the Lorenz system [1,2], different types of chaotic systems that exhibited research and application prospect have been discovered successively, such as double-scroll family, multi-scroll and grid multi-scroll chaotic system, ring-shaped multi-wing generalized Lorenz system family and so on [3–15]. These discoveries not only have stimulated the depth study of the nonlinear dynamic system theory, but also unfolded a promising prospect of their relevant applications in engineering technologies [15].

It is well known that the main design criterion of conventional approach for generating grid multi-scroll chaotic system lies in the choices of an appropriate double-scroll chaotic system, from which the number of index 2 saddle-focus equilibrium points can be increased and extended along a certain plane or space [4–15]. One may ask whether or not there exists another method to break such a limitation? This Letter gives a positive answer to the question.

To be specific, in this Letter, based on a third-order linear system and equipped with a controller $\boldsymbol{F} = (-f_1(x-y), f_1(x-y), -f_2(z))^T$, a novel approach for creating grid multi-scroll chaotic attractors is investigated. A third-order linear system is designed to ensure that its unique equilibrium point belongs to a saddle-focus type with index 2 and the corresponding eigenvalues satisfy Shilnikov conditions. The basic working principle of this approach is to divide original third-order linear system into several different linear subspace, each of which having only a saddle-focus equilibrium point with index 2, extending them along both xy plane and z axis direction, so as to generate grid multi-scroll chaotic attractors. Furthermore, basic dynamical behaviors are also investigated, confirming the chaotic nature of the presented system. Based on dimensionless state equations and modified module-based method, the success of design has been verified by circuit implementations, which is in good agreement with numerical simulations.

The organization of the Letter is as follows. A third-order linear system is introduced in Section 2. The generation of grid multi-scroll chaotic attractors via switching control is investigated in Section 3. The basic dynamical behaviors for grid multi-scroll chaotic attractors are analyzed in Section 4. A module-based circuit design and circuit implementation observations are demonstrated in Section 5. Finally, conclusions are given in Section 6.

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2. Design of a third-order linear system

In this section, a third-order linear system is designed. The characteristic of this designed system is that the Jacobin matrix is full rank to ensure that there exists a unique equilibrium point which belongs to saddle-focus type with index 2. And its corresponding eigenvalues satisfy Shilnikov conditions. Consider the following third-order linear system:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = J \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

If rank$[J] = 3$, the only equilibrium point of system (1) is $O(0, 0, 0)$, and its characteristic polynomial of the Jacobian matrix J is:

$$\lambda^3 + A\lambda^2 + B\lambda + C = 0 \tag{2}$$

Letting eigenvalues of Eq. (2) be λ_1, λ_2, λ_3, respectively, one can get the coefficients A, B, C, given by:

$$\begin{cases} A = -(a_{11} + a_{22} + a_{33}) = -(\lambda_1 + \lambda_2 + \lambda_3) \\ B = a_{22}a_{33} - a_{23}a_{32} + a_{11}a_{33} - a_{13}a_{31} + a_{11}a_{22} - a_{12}a_{21} = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_3\lambda_2 \\ C = -a_{11}a_{22}a_{33} + a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33} - a_{12}a_{23}a_{31} - a_{13}a_{21}a_{32} + a_{13}a_{31}a_{22} = -\lambda_1\lambda_2\lambda_3 \end{cases} \tag{3}$$

substituting $\lambda = \mu - A/3$ into Eq. (2) yields:

$$\mu^3 + P\mu + Q = 0 \tag{4}$$

where $P = -A^2/3 + B$, $Q = 2A^3/27 - AB/3 + C$.

Eq. (4) can be solved by using the Cardan formula. For example, letting $\Delta = 4P^3 + 27Q^2$, if $\Delta > 0$, Eq. (4) has a unique negative real eigenvalue γ_1 and a pair of complex conjugate eigenvalues $\sigma_1 \pm j\omega_1$, given by:

$$\begin{cases} \gamma_1 = \sqrt[3]{-\frac{Q}{2} + \sqrt{\frac{\Delta}{4\times27}}} + \sqrt[3]{-\frac{Q}{2} - \sqrt{\frac{\Delta}{4\times27}}} \\ \sigma_1 = -\frac{1}{2}\left(\sqrt[3]{-\frac{Q}{2} + \sqrt{\frac{\Delta}{4\times27}}} + \sqrt[3]{-\frac{Q}{2} - \sqrt{\frac{\Delta}{4\times27}}}\right) \\ \omega_1 = \frac{\sqrt{3}}{2}\left(\sqrt[3]{-\frac{Q}{2} + \sqrt{\frac{\Delta}{4\times27}}} - \sqrt[3]{-\frac{Q}{2} - \sqrt{\frac{\Delta}{4\times27}}}\right) \end{cases} \tag{5}$$

Referring to Eqs. (4) and (5), the three eigenvalues of Eq. (2) can be solved by

$$\begin{cases} \lambda_{1,2} = \sigma \pm j\omega = (-A/3 + \sigma_1) \pm j\omega_1 \\ \lambda_3 = \gamma = -A/3 + \gamma_1 \end{cases} \tag{6}$$

In Eq. (6), if $\gamma < 0$, $\sigma > 0$, $|\sigma/\gamma| < 1$, the equilibrium point $O(0, 0, 0)$ of system (1) is saddle-focus one with index 2, and the eigenvalues satisfy the Shilnikov conditions. Among them, the negative real eigenvalue is associated with one-dimensional stable manifold, whereas a pair of complex conjugate eigenvalues with two-dimensional unstable manifold.

Letting $a_{11} = a_{12} = a_{21} = a_{23} = a_{32} = 0$, $a_{13} = 8$, $a_{22} = -6$, $a_{31} = -8$, $a_{33} = 0.6$, system (1) becomes:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A_0 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \Rightarrow \quad \dot{\boldsymbol{X}} = A_0\boldsymbol{X} \tag{7}$$

where $\dot{\boldsymbol{X}} = (\dot{x}, \dot{y}, \dot{z})^T$, $\boldsymbol{X} = (x, y, z)^T$. According to the above criterion, one gets $A = -(a_{22} + a_{33}) = 5.4$, $B = a_{22}a_{33} - a_{13}a_{31} = 60.4$, $C = a_{13}a_{31}a_{22} = 384$, $P = -A^2/3 + B = 50.68$, $Q = 2A^3/27 - AB/3 + C = 286.944$, $\Delta = 4P^3 + 27Q^2 = 2.7 \times 10^6 > 0$, $\gamma = -6$, $\sigma \pm j\omega = 0.3 \pm j7.9944$. Since $\gamma < 0$, $\sigma > 0$ and $|\sigma/\gamma| < 1$, the eigenvalues of the equilibrium point $O(0, 0, 0)$ satisfy the Shilnikov conditions.

3. Generation of grid multi-scroll chaotic attractors via switching control

Based on system (7), the switching controllers $f_1(x+y)$ and $f_2(z)$ are introduced to generate various grid multi-scroll chaotic attractors:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix} \begin{pmatrix} -f_1(x-y) \\ f_1(x-y) \\ -f_2(z) \end{pmatrix} \quad \Rightarrow \quad \dot{\boldsymbol{X}} = A_0(\boldsymbol{X} + \boldsymbol{F}) \tag{8}$$

where $\dot{\boldsymbol{X}} = (\dot{x}, \dot{y}, \dot{z})^T$, $\boldsymbol{X} = (x, y, z)^T$, $\boldsymbol{F} = (-f_1(x-y), f_1(x-y), -f_2(z))^T$ are switching controllers, which is used to divide original linear system (7) into several different linear subspaces V_i $(i = 0, \pm1, \pm2, \ldots)$, each of which having only a equilibrium point with index 2. $f_1(x-y)$ and $f_2(z)$ can extend the saddle-focus equilibrium points with index 2 along both xy plane and z axis direction, respectively. Therefore, various grid multi-scroll chaotic attractors can be created in system (8). One can consider the controllers as below:

$$\begin{cases} f_1(x-y) \in \begin{pmatrix} f_{11}(x-y) \\ f_{12}(x-y) \end{pmatrix} \\ f_2(z) \in \begin{pmatrix} f_{21}(z) \\ f_{22}(z) \end{pmatrix} \end{cases} \tag{9}$$

In the following, three different types of control strategies are investigated for generating various grid multi-scroll chaotic attractors.