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Sliding in a piecewise-smooth dynamical system with a hold-on effect

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ABSTRACT

In this paper, we study a piecewise-smooth dynamical system of a Filippov type that has a specific discontinuity in the form of a hold-on term. The system resembles a driven dry-friction oscillator with the difference that the magnitude of the friction depends on the value of a local maximum of the oscillator displacement. Due to the hold-on term, the definition of the sliding region is not trivial. Along with a sliding region that is found analytically and is valid for any orbits, there exists a sliding region that we name 'virtual' because it is specific for each particular orbit. The existence of the virtual sliding region is explained by the specific discontinuity — the hold-on term — and the behavior associated with the sliding region and its boundaries can be considered as a new type of sliding-associated behavior.

1. Introduction

Systems described by nonsmooth (or piecewise-smooth) vector fields or discrete-time equations are investigated in the framework of nonlinear dynamics [1–3]. Nonsmooth systems, both continuous- and discrete-time, model many realistic systems: mechanical impact [4] and dry-friction oscillators [5–7], hybrid systems [8,9], sigma–delta modulators [10], DC–DC converters [11,12], microelectromechanical systems (MEMS) [13,14] and superconductive resonators [15]. Nonsmooth systems give rise to a number of complex phenomena that are not found in smooth systems [16–20] and require a new approach to classification [21] and numerical modeling [9,22,23].

In this paper we study a piecewise-smooth electronic oscillator that models an electrostatic vibration energy harvester [24–26], a device that converts kinetic energy of ambient vibrations into electricity. This device contains both mechanics (a high-Q resonator) and conditioning electronics, coupled together through a variable capacitor (a transducer). The circuit generates a piecewise smooth nonlinear damping force and influences the dynamics of the resonator. The interest in the dynamics of these systems is driven by practical problems. The desired mode of the oscillator is a steady-state harmonic regime, but the oscillator can also display very complex behavior and bifurcations typical of both smooth and nonsmooth systems [27,28]. A mathematical model of the system takes the form of nonlinear piecewise-smooth differential equations [25,26], namely Filippov systems, with a discontinuity appearing due to the conditioning electronics.

In our previous works, we have investigated steady-state oscillations [25,26] and develop methods that can be used to describe steady-state oscillations in electrostatic oscillators of this type. In study [27], we gave preliminary results on chaotic behavior. The limit on converted power and optimization problem is discussed in [29]. Despite seeming simplicity, the model studied in the paper can demonstrate a broad range of nonlinear behavior associated with both smooth and piecewise smooth dynamical systems. In this study, we investigate specific behavior associated with sliding that is displayed due to both the non-smoothness of the vector field governing the system dynamics and a hold-on term.

Indeed, the equation that governs the dynamics of the electronic oscillator resembles a dry-friction oscillator and includes a term that can be seen as electrical damping that is defined by the sign of the velocity. This term introduces the "classic" non-smoothness of the vector field. On the other hand, this electrical damping depends on the local maximum value of the resonator displacement. Over the time interval until the next local maximum is not reached, the electric damping term does not change. However, if a new local maximum is reached, the damping term changes. Therefore, we have a type of a non-smooth nonlinear system with a hold-on effect. Apart from a normal sliding region whose boundaries can be determined using the approach described, for instance, in [2], the hold-on effect results in a piecewise-defined boundary of a sliding region. It is a novel type of behavior and we denote this boundary and the sliding region as 'virtual' because they are specifically defined for each orbit. In this paper we discuss some specific dynamics associated with this

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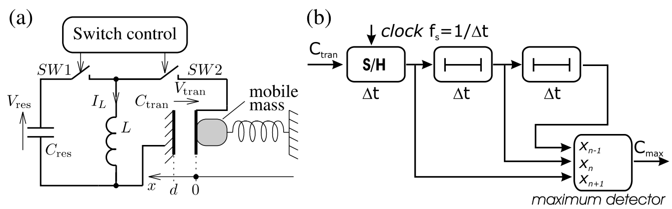

Fig. 1. (a) Scheme of the electronic oscillator that models a vibration electrostatic energy harvester. (b) Scheme of a digital detector of local maxima. It samples a signal with a very high frequency and hold three values. By comparing the middle value with the others, it detects an event (peak of the signal).

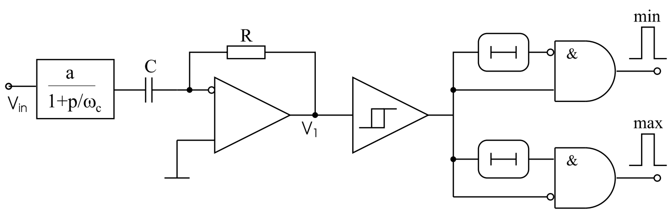

Fig. 2. Scheme of an analog detector of local maxima/minima (analog peak detector). It contains a capacitor, a resistor, an operational amplifier, a comparator with hysteresis and a signal filter to remove noise. The output of the comparator is hold and compared with the current value.

phenomenon: the collision of orbits with the virtual sliding region. Virtual sliding boundaries and sliding regions will also exist in a similar system employing nonlinear resonators [28].

2. Statement of the problem

The electronic oscillator under investigation consists of a high-Q linear resonator, a variable capacitor (transducer) C_{tran} and a conditioning circuit as shown in Fig. 1. A detailed description of the conversion cycle and a schematic view of the system can be found in [25,26]. Briefly, the operation of the electronic oscillator can be described as follows: a resonator oscillates as it is driven by ambient vibration and, since it is attached to a transducer, this causes changes of the transducer capacitance $C_{\text{tran}}(t)$ with time. The conditioning circuit determines a local maximum of the time-varying capacitance and charges it at this moment. When the transducer capacitance decreases, it has a charge given by the conditioning circuit. Therefore, the conditioning circuit generates an electrostatic force that acts on the mechanical resonator together with external driving. When the transducer capacitance increases, there is no charge on the transducer, and the electrostatic force is absent. Hence, the force is a piecewise defined function: $f_t = 0$ if $\dot{x} > 0$ and $f_t \neq 0$ otherwise, and will be referred later as $f_t(x, \dot{x})$. Here x is the dimensional displacement of the resonator.

The normalized displacement $y = x/d$ of the resonator can be described by a second-order ordinary differential equation

$$y'' + 2\beta y' + y = \alpha \cos(\Omega\tau) + f_t(y, y') \tag{1}$$

where the prime denotes the derivative with respect to normalized time τ. The normalized parameters are: time $\tau = \omega_0 t$, the dissipation $\beta = b/(2m\omega_0)$, the external frequency $\Omega = \omega_{\text{ext}}/\omega_0 = 1 + \sigma$ (σ is a possible small mismatch between the two frequencies) and external acceleration amplitude $\alpha = A_{\text{ext}}/(d\omega_0^2)$. The dimensional parameters used for the normalization are as follows: m is the mass of the resonator, b is the damping factor, $\omega_0 = \sqrt{k/m}$ is the natural frequency, ω_{ext} is the external frequency and A_{ext} is the external acceleration amplitude.

The conditioning circuit operates the switches from Fig. 1a using a detector of local maxima/minima (peak detector). It can be either a digital detector (shown in Fig. 1b) or analog (shown in Fig. 2). The performance of the digital peak detector is described as follows. This detector has a sampling block (with a very high sampling frequency f_s) and a block that can store previous values of the capacitance. The detector of local maxima analyses the three recent values of the transducer capacitance C_{tran} and if the middle value of C_{tran} is greater than the two other values, it detects an event (a local maximum). Since $C_{\text{tran}} = C_0/(1 - y(\tau))$ is a smooth function of the displacement $y(\tau)$, where C_0 is the capacitance of the transducer if no force is applied, finding a local maximum or minimum corresponds to finding zeroes of the velocity y'.

An analog peak detector operates as follows. An analog differentiator implemented with an operational amplifier calculates the derivative of the input signal. At local maxima at the input, the voltage V_1 goes from positive to negative values. The zero-crossing of V_1 is detected by a comparator: its output goes from 0 to 1. The comparator is provided with a hysteresis characteristic in order to additionally filter the noise. In a similar manner, the minima at the input are detected by the circuit, and the comparator output goes from 1 to 0. Realistic analog peak detectors also include a filter that can additionally cut off high-frequency noise (see Fig. 2). The analog peak detector model used for simulations presented in this paper includes both, a signal filter for noise pre-filtering and a comparator with hysteresis to further diminish the effect of noise.

The subsequent conditioning circuit from Fig. 1a processes the information about the max/min events, for instance, by generating a small pulse for switches *SW1* and *SW2*. We will consider the digital peak detector as the main case in the paper (most of results will be presented for this case), however, we discuss briefly the difference in the dynamics of the system for both types of peak detectors. We mentioned these two cases since this is an event-driven system and the peak detector defines the event. Since there is a difference in the operation of the two peak detectors, the dynamics of the system can be different in these two cases.

Now we can write the function f_t, the normalized transducer force:

$$f_t(y, y') = \begin{cases} \frac{\nu_W}{1 - y_{\max}}, & y' < 0 \\ 0, & y' > 0 \end{cases} \tag{2}$$

where $\nu_W = W_0/(d^2 m\omega_0^2)$ and W_0 is the value of energy fixed at the transducer at each local maximum of $y(\tau)$ (or equivalently of $C(\tau)$). We define $y_{\max}$ as the value of the displacement when the peak detector detects an event (a local maximum). In addition, we note that a small delay $\Delta\tau$ in the definition of a local maximum may exist. For instance, in the case of the digital peak detector, this delay $\Delta\tau$ is defined by the sampling time $T_s = 1/f_s$. In the case of the analog peak detector, the delay is defined by the bandwidth of the detector.

More discussion on the architecture of the conditioning circuit and on optimal regimes of circuit operation can be found in [25,26,29]. For further numerical simulations we use the parameter values proposed in [26]. For a particular system (1) and (2), ν_W and α can change while the other parameters are typically fixed.

3. Normal form of the system and the boundaries of the sliding region

Eqs. (1) and (2) can be written in the standard form $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$ if we introduce the vector $\mathbf{x} = (x_1, x_2, x_3)$ where $x_1 = y$, $x_2 = y'$ and $x_3 = \tau$:

$$\dot{\mathbf{x}} = \begin{cases} \mathbf{F}_1(\mathbf{x}), & H(\mathbf{x}) > 0 \\ \mathbf{F}_2(\mathbf{x}), & H(\mathbf{x}) < 0 \end{cases} \tag{3}$$

The vector functions $\mathbf{F}_{1,2}$ are

$$\mathbf{F}_i(\mathbf{x}, \mathbf{P}) = \begin{cases} x_2 \\ -2\beta x_2 - x_1 + \alpha \cos\Omega x_3 + a_i f_t, & i = 1, 2 \\ 1 \end{cases} \tag{4}$$