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Elliptic waves in two-component long-wave–short-wave resonance interaction system in one and two dimensions

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ABSTRACT

We consider $(2+1)$- and $(1+1)$-dimensional long-wave–short-wave resonance interaction systems. We construct an extensive set of exact periodic solutions of these systems in terms of Lamé polynomials of order one and two. The periodic solutions are classified into three categories as similar, mixed, superposed elliptic solutions. We also discuss the hyperbolic solutions as limiting cases.
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1. Introduction

The study of nonlinear waves is of broad scientific interest [1]. Nonlinear waves in multi component long-wave–short-wave resonant interaction (LSRI) system have received significant attention in recent years. Here nonlinear resonance interaction between a low frequency long-wave (LW) and multiple high frequency short waves (SWs) takes place when the phase velocity (say v_p) of the former exactly or approximately matches with the group velocity (say v_g) of the short waves, that is, $v_p \simeq v_g$. This LSRI phenomenon has a wide range of applications ranging from water waves to nonlinear optics which also include biophysics and plasma physics. In the SW components the soliton is formed due to a delicate balance between the dispersion and the nonlinear interaction of LW with the SWs while in the LW component, the soliton formation is determined solely by the self-interaction of short-wave packets.

The pioneering work of LSRI system was done by Zakharov [2]. Later on, the general Zakharov equations in one dimension have been reduced to the integrable Yajima–Oikawa system in Ref. [3]. At the same time, independently Benney has derived the model equation for the interaction of short wind-driven capillary gravity wave in deep water [4]. The experimental study of LSRI in a three-layer fluid was carried out by Kopp and Redekopp [5]. Then, in a physical setup of two-layer fluid model the one- and two-dimensional LSRI systems have been derived and bright and dark soliton solutions have also been obtained in Refs. [6,7].

Recently, Kanna et al. have shown that the following $(1+1)$-dimensional (i.e., one time and one space dimensions) LSRI system [8]

$$iS_{j,t} + \delta S_{j,xx} + LS_j = 0, \quad j = 1, 2, \tag{1a}$$

$$L_t = 2\sum_{j=1}^{2} c_j |S_j|^2_x, \tag{1b}$$

can be deduced from a set of three coupled nonlinear Schrödinger equations governing the propagation of three optical fields in a triple mode optical fiber, by applying the asymptotic reduction procedure. In Eq. (1), S_j and L, respectively, indicate jth SW and (one) LW, t and x represent the partial derivatives with respect to evolutional and spatial coordinates, respectively, and the nonlinearity coefficients c_j, $j = 1, 2$, are arbitrary real parameters. Here $\delta = \pm 1$ and for $\delta = 1$ the above system (1) is nothing but the two-component Yajima–Oikawa (YO) system. Eq. (1) also appears in the study of interaction of quasi-resonant two-frequency SW pulses

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with a LW [9]. Such multi-component YO system also has been derived in the context of multiple component magnon–phonon system [10]. In Ref. [8], we have obtained the bright n-soliton solution of the above system (1) and have revealed the fact that the bright solitons can undergo two types of fascinating energy sharing collisions. Here the presence of the LW induces nonlinear interaction between two SWs which leads to nontrivial collision behaviour. The rogue waves of LSRI system with $j=1$ (one SW and LW components) and $\delta=1$ have been reported in Ref. [11].

The two-dimensional multi-component LSRI system has also received equally good attention as that of their one-dimensional counterpart. Particularly the following two-component analogue of the $(2+1)$-dimensional (i.e., two spatial coordinates x, y and one time coordinate) LSRI system

$$i[S_{j,t}+\varepsilon_j S_{j,y}]+\delta S_{j,xx}+LS_j=0, \quad j=1,2, \tag{2a}$$

$$L_t=2\sum_{j=1}^{2} c_j|S_j|_x^2, \tag{2b}$$

where the subscripts x and y represent the partial derivatives with respect to spatial coordinates and t represents temporal coordinate, the nonlinearity coefficients c_1 and c_2 and the coefficient ε_j, $j=1,2$, are real arbitrary parameters and $\delta=\pm 1$. Eq. (2) has been derived as the governing equation for the interaction of three nonlinear dispersive waves in optical fiber or in photorefractive medium by applying a reductive perturbation method [12]. In the above system, two SWs propagate in anomalous dispersion regime and the real LW propagates in the normal dispersion regime. In a recent work [13], Kanna et al. have generalized the approach of [12] and derived a M-component LSRI system as the propagation equation for multiple dispersive waves (say $(M+1)$ waves) in a weak Kerr type nonlinear medium in the small amplitude limit. To get further physical insight into the above system (2), we would like to point out that the one-component $(j=1)$ version of Eq. (2) can be derived from the governing equation for two-dimensional two-wave interaction [14,15] by following the approach of [12]. Thus system (2) is a three-wave generalization of SW system in $(2+1)$ dimensions. From a mathematical perspective, the soliton solutions of system (2), with $c_1=c_2=1$ are constructed in Refs. [12,16]. Particularly, in Ref. [16] it has been shown that the bright solitons exhibit interesting energy sharing collisions characterized by intensity (energy) redistribution, amplitude dependent phase-shifts and change in relative separation distances.

Periodic nonlinear waves can also arise in real physical systems. For example, generation of ultrashort pulse-train by using nonlinear transform of a twin frequency signal is one such real system arising in nonlinear optics ([17] and references therein). Thus to describe real situations one may need special type of periodic solutions. Several periodic solutions of integrable and nonintegrable multicomponent nonlinear Schrödinger equations with focusing, defocussing and mixed type nonlinear interactions have been obtained in Refs. [17–26], in terms of Jacobi elliptic functions. So far, such elliptic wave solutions have not been constructed for the one- and two-dimensional two-component LSRI systems (1) and (2) as these integrable systems have been reported recently. This paper is aimed at constructing different families of elliptic wave solutions of (1) and (2) in a systematic way.

The organization of the paper is as follows. In Section 2, the elliptic wave solutions of the $(2+1)$-dimensional two-component LSRI system (2) are obtained in terms of Lamé polynomials of orders one and two. Similar solutions of $(1+1)$-dimensional two-component LSRI system (1) are discussed in Section 3. Finally, conclusions are drawn in the last section.

2. Jacobi elliptic function solutions of the $(2+1)$-dimensional two-component LSRI system

We start with the $(2+1)$-dimensional LSRI system (2). To construct the Jacobi elliptic solutions of Eq. (2a) we choose the traveling wave ansatz

$$S_j(x,y,t)=f_j\big[\beta(x-vt-wy+\delta_0)\big]e^{-i(\omega_j t+\nu_j y-k_j x+\delta_j)}, \quad j=1,2. \tag{3}$$

Here f_j are real functions of x, y and t; β, δ_0 and $\delta_{1,2}$ are real constants, ω_j is the frequency of the jth SW component, k_j is the wave number, v is the velocity, w and ν_j are real parameters. Note that both the SWs are traveling with the same velocity. Inserting the above ansatz (3) into Eq. (2b), we obtain the LW component as

$$L=-\frac{2}{v}\big(c_1|S_1|^2+c_2|S_2|^2\big). \tag{4}$$

Following this, by substituting the ansatz (3) into (2) and also by using (4), we get a set of complex equations. On equating the real and imaginary parts, we respectively obtain

$$\frac{d^2 f_j}{du^2}+\left[\frac{\delta(\omega_j+\varepsilon_j\nu_j)-k_j^2}{\beta^2}-\frac{2\delta}{v\beta^2}\big(c_1 f_1^2+c_2 f_2^2\big)\right]f_j=0, \quad j=1,2, \tag{5a}$$

$$v+\varepsilon_j w=2\delta k_j, \tag{5b}$$

where $u=\beta(x-vt-wy+\delta_0)$. At a first look it might seem that (5) is similar to the coupled nonlinear Schrödinger system given in [19]. But a careful analysis shows that they are essentially different. This is due to the presence of the LW component L. In fact, here the solution parameters β and the velocity v explicitly appear before the non-linear term $(c_1 f_1^2+c_2 f_2^2)$. This makes the present system different from that of [19]. Particularly, this v determines the nature of the solution, i.e., whether the solution is singular or not, as will be shown later. Thus the results presented here are distinct from those given in [19], though the elliptic function solutions take standard Lamé function profiles as will be demonstrated below. These solutions can be viewed as velocity locked solutions.

Next, we assume the Lamé function ansatz for f_j, that is,

$$f_j=\rho_j\psi_j^{(l)}, \quad l,j=1,2, \tag{6}$$

where $\psi_j^{(l)}$ can be anyone of the three first order Lamé polynomials for $l=1$ and for $l=2$, it can be any one of the five second order Lamé polynomials and satisfy the Lamé equation [27],

$$\frac{d^2\psi_j^{(l)}}{du^2}+\big[\lambda_j^{(l)}-l(l+1)m\,\mathrm{sn}^2(u,m)\big]\psi_j^{(l)}=0, \tag{7}$$

where m $(0\le m\le 1)$ is the modulus parameter of the Jacobi elliptic function $\mathrm{sn}(u,m)$, l $(=1,2)$ represents the order of the Lamé polynomial $\psi_j^{(l)}$ and $\lambda_j^{(l)}$ is the corresponding eigenvalue. Thus we will have two distinct families of solutions corresponding to the Lamé polynomials of order 1 $(l=1)$ and of order 2 $(l=2)$. First, we present and discuss periodic solutions in terms of Lamé polynomials of order one and then we present the second order solutions.

2.1. Solutions in terms of Lamé polynomials of order 1

The two-component LSRI system (2) admits seven distinct periodic solutions in terms of Lamé polynomials of order 1. These first order solutions of Eq. (2) corresponding to $l=1$ can be expressed in terms of Jacobi elliptic functions [28]. We classify these solutions as similar, mixed and superposed elliptic solutions. By similar we mean same kind of standard elliptic function profile for both

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