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Randomness control of vehicular motion through a sequence of traffic signals at irregular intervals

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ABSTRACT

We study the regularization of irregular motion of a vehicle moving through the sequence of traffic signals with a disordered configuration. Each traffic signal is controlled by both cycle time and phase shift. The cycle time is the same for all signals, while the phase shift varies from signal to signal by synchronizing with intervals between a signal and the next signal. The nonlinear dynamic model of the vehicular motion is presented by the stochastic nonlinear map. The vehicle exhibits the very complex behavior with varying both cycle time and strength of irregular intervals. The irregular motion induced by the disordered configuration is regularized by adjusting the phase shift within the regularization regions.

1. Introduction

Physics, other sciences and technologies meet at the frontier area of interdisciplinary research. Transportation problems have been extensively investigated by engineers so far. Recently, traffic problems have attracted much attention in the fields of physics [1–5]. The concepts and techniques of physics are being applied to such complex systems as transportation systems. The traffic flow, pedestrian flow, and bus-route problem have been studied from a point of view of statistical mechanics and nonlinear dynamics [6–15]. The jams, chaos, and pattern formation are typical signatures of the complex behavior of transportation.

Mobility is nowadays one of the most significant ingredients of a modern society. In urban traffic, vehicles are controlled by traffic signals to give priority for a road and to insure road safety because they encounter at crossings. In real traffic, the vehicular traffic depends highly on the control of traffic signals. Until now, one has studied the periodic traffic controlled by a few traffic signals. It has been concluded that the periodic traffic does not depend on the number of traffic lights [16,17]. Very recently, a few works have been done for the traffic of vehicles moving through an infinite series of traffic signals with the same interval. The effect of cycle time on vehicular traffic has been clarified [18–22]. Also, Lammer and Helbing have studied the effect of self-controlled signals on vehicular flow based on fluid-dynamic and many-particle simulations [23].

Generally, the traffic lights are controlled by either synchronized or delayed strategies. In the synchronized strategy, all the signals change simultaneously and periodically where the phase shift has the same value for all signals. In the delayed strategy, the signal changes with a certain time delay between the signal phases of two successive intersections. The delayed strategy is called the green-wave strategy because the red signal changes successively the green from the upstream to downstream (or from the downstream to the upstream) with a constant value of phase difference. The change of traffic lights propagates backward like a green wave. Thus, the vehicular traffic depends highly on the signal's strategy. The operator will be able to control the traffic signal by the use of the other strategy. Specifically, one can manage both cycle time and phase shift of signals.

The vehicular traffic through a series of signals positioned at regular intervals has been studied. However, the signal's intervals vary from position to position in real traffic. The disordered configuration of signals has an important effect on the control of vehicular traffic because the signal disorder induces an irregular motion of vehicles [24]. In real traffic, it is very important to control the irregular motion. However, the control of the irregularity by signals has been little investigated until now.

In this Letter, we study the randomness control for the vehicular traffic through a series of traffic signals positioned at irregular intervals. We control the traffic light by both cycle time and phase shift. The delay or advance of arrival time induced by the irregular interval is compensated by adjusting the phase shift. We present the stochastic nonlinear dynamic model for the vehicular motion through the sequence of traffic signals with the disordered configuration. We investigate the dynamical behavior of a single vehicle.

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We clarify the randomness control and dynamical behavior of a single vehicle through the sequence of signals by varying both cycle time and phase shift.

2. Stochastic nonlinear model

The traffic signals are positioned at irregular intervals on a roadway. We consider the motion of a single vehicle going through the series of traffic signals. The traffic signals are numbered, from upstream to downstream, by $1, 2, 3, \ldots, n, n+1, \ldots$. The interval between signals n and $n+1$ is indicated by $l(n)$. All the signal changes periodically with period t_s. Period t_s is called as the cycle time. The phase shift of signals varies from signal to signal. The phase shift of signal n is represented by $t_{phase}(n)$. The vehicle moves with the mean speed v between a traffic signal and its next signal.

In the synchronized strategy, all the traffic signals change simultaneously from red (green) to green (red) with a fixed time period $(1-s_p)t_s$ $(s_p t_s)$. The period of green is $s_p t_s$ and the period of red is $(1-s_p)t_s$. Fraction s_p represents the split which indicates the ratio of green time to cycle time.

The signal timing is controlled by offset time t_{offset}. The offset time means the difference of phase shifts between two successive signals. In the delayed (green wave) strategy, the phase shift of signal n is given by $t_{phase}(n) = nt_{offset}$. Then, the signal switches from red to green in green wave way. Here, we control the signals by adjusting the phase shift. Phase shift $t_{phase}(n)$ is synchronized with signal's interval $l(n)$. The phase shift changes irregularly from signal to signal similarly to the signal's interval.

When a vehicle arrives at a traffic signal and the traffic signal is red, the vehicle stops at the position of the traffic signal. Then, when the traffic signal changes from red to green, the vehicle goes ahead. On the other hand, when a vehicle arrives at a traffic signal and the traffic signal is green, the vehicle does not stop and goes ahead without changing speed.

We define the arrival time of the vehicle at traffic signal n as $t(n)$. The arrival time at traffic signal $n+1$ is given by

$$\begin{aligned} t(n+1) = {} & t(n) + l(n)/v \\ & + \big(r(n) - t(n)\big) H\big(t(n) + t_{phase}(n) \\ & - \big\{\mathrm{int}\big[\big(t(n) + t_{phase}(n)\big)/t_s\big]\big\} t_s - s_p t_s\big) \end{aligned} \tag{1}$$

with

$$r(n) = \big\{\mathrm{int}\big[\big(t(n) + t_{phase}(n)\big)/t_s\big] + 1\big\} t_s - t_{phase}(n),$$

where $H(t)$ is the Heaviside function: $H(t) = 1$ for $t \geqslant 0$ and $H(t) = 0$ for $t < 0$. $H(t) = 1$ if the traffic signal is red, while $H(t) = 0$ if the traffic signal is green. $l(n)/v$ is the time it takes for the vehicle to move between signals n and $n+1$. $r(n)$ is such time that the traffic signal just changed from red to green. The third term on the right-hand side of Eq. (1) represents such time that the vehicle stops if traffic signal n is red. The number n of iteration increases one by one when the vehicle moves through the traffic signal. The iteration corresponds to the going ahead on the highway. Signal's interval $l(n)$ and phase shift $t_{phase}(n)$ are irregular (random) variables. Eq. (1) is a stochastic nonlinear dynamics.

We consider the control of signals by adjusting the phase shift. When a vehicle passes through signal n, it arrives at signal $n+1$ after tour time $l(n)/v$. This tour time varies from signal to signal with interval $l(n)$. One can compensate time difference $(l(n) - \langle l \rangle)/v$ by adjusting phase shift $t_{phase}(n)$ where $\langle l \rangle$ is the mean value of interval $l(n)$. Phase shift $t_{phase}(n)$ is synchronized with irregular interval $l(n)$ as follows

$$t_{phase}(n) = \big(l(n) - \langle l \rangle\big)/v. \tag{2}$$

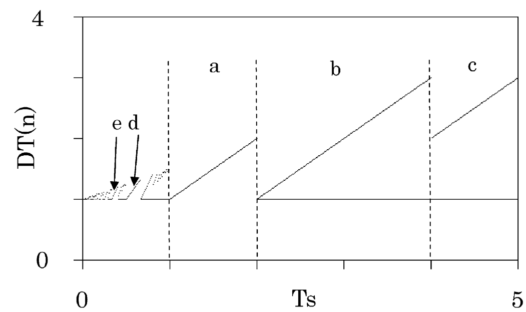

Fig. 1. Plot of tour time $DT(n)$ against cycle time T_s from $n = 500$ to $n = 1000$ for the vehicular traffic with signals positioned at regular intervals.

Then, time difference $(l(n) - \langle l \rangle)/v$ is compensated by the phase shift. It is expected that the irregular motion induced by the disordered configuration will be regularized by the adjustment of phase shift.

By dividing time by the characteristic time $\langle l \rangle/v$, one obtains the stochastic nonlinear equation of dimensionless arrival time:

$$\begin{aligned} T(n+1) = {} & T(n) + l(n)/\langle l \rangle \\ & + \big(R(n) - T(n)\big) H\big(T(n) + T_{phase}(n) \\ & - \big\{\mathrm{int}\big[\big(T(n) + T_{phase}(n)\big)/T_s\big]\big\} T_s - s_p T_s\big) \end{aligned} \tag{3}$$

with

$$R(n) = \big\{\mathrm{int}\big[\big(T(n) + T_{phase}(n)\big)/T_s\big] + 1\big\} T_s - T_{phase}(n),$$

where $T(n) = t(n)v/\langle l \rangle$, $R(n) = r(n)v/\langle l \rangle$, $T_{phase}(n) = t_{phase}(n)v/\langle l \rangle$ and $T_s = t_s v/\langle l \rangle$. The dimensionless form of Eq. (2) is given by

$$T_{phase}(n) = l(n)/\langle l \rangle - 1. \tag{4}$$

The dynamics of the vehicle is described by the stochastic nonlinear map (3) with Eq. (4) [25]. The vehicular motion will exhibit a complex behavior with varying the cycle time. We study how the irregular motion is controlled by varying both cycle time T_s and phase shift $T_{phase}(n)$.

For the signals positioned at the regular intervals, the map (3) has the following properties. For $1 < T_s \leqslant 2$, the difference $T(n+1) - T(n)$ has a constant value. For $2 < T_s \leqslant 4$, the difference displays the periodic behavior with period 2. For $4 < T_s \leqslant 6$, the difference displays the periodic behavior with period 3. For $2k < T_s \leqslant 2(k+1)$ where k is an integer equal to 1 or larger than 1, the difference displays the periodic behavior with period $k+1$. For $1/2 < T_s \leqslant 2/3$, the difference has a constant value. When T_s increases from $2/3$ to 1, the periodic behavior of difference varies successively from period 2 to period infinity. The points of period infinity appear at $T_s = 1/k$ where k is an integer larger than 1.

We do not derive the features of map (3) analytically but we obtain the following property from Fig. 1 graphically. The regions of cycle time T_s in which difference $T(n+1) - T(n)$ has a constant value are given by $1 < T_s \leqslant 2, 1/2 < T_s \leqslant 2/3, 1/3 < T_s \leqslant 2/5, \ldots, 1/k < T_s \leqslant 2/(2k-1), \ldots$ where k is an integer larger than 1.

3. Simulation result

We investigate the vehicular motion through the series of traffic signals by iterating stochastic nonlinear map (3). We calculate the arrival time at traffic light n when the vehicle goes ahead through the series of traffic signals with the disordered configuration. We study how the tour time varies with cycle time and strength of irregularity. We clarify the dynamical behavior of the signal traffic controlled by adjusting the phase shift.