Physics Letters A 372 (2008) 4793–4798

Contents lists available at ScienceDirect

Physics Letters A

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Self-similar structures in a 2D parameter-space of an inductorless Chua's circuit

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ARTICLE INFO

Article history:
Received 7 February 2008
Received in revised form 7 April 2008
Accepted 1 May 2008
Available online 21 May 2008
Communicated by A.R. Bishop

PACS:
05.45.Ac
05.45.Pq

Keywords:
Inductorless Chua's circuit
Parameter-space
Shrimps
Self-similarities
Lyapunov exponents

ABSTRACT

In a 2D parameter-space of an inductorless Chua's circuit model, we carried out numerical investigations and observed self-similar stability structures embedded in a sea of chaos, known until recently just in discrete-time models, namely, shrimps. We showed that those structures are self-similar and organize themselves in a period-adding bifurcation cascade in a region of the parameter-space.

1. Introduction

Parameter-spaces of dynamical systems have been studied extensively in discrete-time models [1] and recently great interests emerged in the parameter-space of continuous-time models [2–5]. Gallas [1] observed periodic structures embedded in chaotic sea, known as shrimps, in the parameter-space of the Hénon map. Those structures organize themselves along very specific directions in parameter-space. Recently, Bonatto et al. [2,3] observed the same structures in the parameter-space of the CO_2 and semiconductor laser models. They showed that the frequency-amplitude space of the CO_2 laser model [2] and the parameter-space of the Hénon map present isomorphisms related with the organization of shrimps in the parameter-spaces. In Ref. [3] they showed, in the parameter-space of the optically injected semiconductor laser model, the presence of regularity structures inside the chaotic phases of the laser model. They observed accumulations of periodic structures embedded inside a chaotic phase and period-adding cascades in the parameter-space.

Experimentally or theoretically, the Chua's circuit [6] has been studied in many applications such as chaos control [7,8] and synchronization [9,10]. That system presents the basic characteristic to compute the parameter-spaces to observe the periodic structures since it has some parameters that control the dynamics of the system. Maranhão et al. [11] illustrated in a simple way the 2D parameter-space of an experimental Chua's circuit and found a chaotic structure, called chaotic fiber, parallel to a period-3 window observed in the experimental bifurcation diagram of the Chua's circuit [6]. In other paper, Maranhão et al. [12] observed experimentally the existence of complex periodic structures in the 2D parameter-space of a Chua's circuit. This circuit is appropriate to observe structures such as those of Ref. [12] since it has only one positive Lyapunov exponent. For systems with more than one positive Lyapunov exponent self-similar structures would not be verified, at least, in a 2D parameter-space.

In this work, we carried out a detailed numerical investigation of a parameter-space of a modified Chua's circuit model, namely, the resistance (R) inductor-resistance (r_L) space of a dimensionless set of equations of motion that model the inductorless Chua's circuit [13,14]. Our motivations came from the results described above in the parameter-spaces of continuous-time models and from one recent paper [14] where we reported results of a theoretical and experimental time series analysis from equations of motion and from an experimental realization of the inductorless Chua's circuit proposed by Tôrres et al. [13]. Here we numerically show that the parameter-space presents self-

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2008.05.036

similar stability structures embedded in a sea of chaos, called shrimps [1–3]. We also show that those self-similar periodic regions organize themselves in a period-adding bifurcation cascade.

This Letter is organized as follows: Section 2 is devoted to the results obtained from the numerical studies of the parameter-space of the inductorless Chua's circuit model, and the concluding remarks are given in Section 3.

2. Numerical results

In the standard form, the set of equations that theoretically describes the dynamical behavior of the Chua's circuit can be easily obtained and is given by

$$\dot{x} = \frac{dx}{dt} = (y - x)/(RC_1) - i_d(x)/C_1,$$
$$\dot{y} = \frac{dy}{dt} = (x - y)/(RC_2) + z/C_2,$$
$$\dot{z} = \frac{dz}{dt} = -y/L - z(r_L/L), \tag{1}$$

where

$$i_d(x) = m_o x + \frac{1}{2}(m_1 - m_0)\big(|x + B_p| - |x - B_p|\big),$$

and R, C_1, C_2, and L are passive linear elements; r_L is the inductor-resistance; i_d is the piecewise linear current through Chua's diode; B_p, m_0, and m_1 are parameters.

In the inductorless implementation, the inductor is replaced by a network, combining operational amplifiers, resistors and capacitors that simulate an inductor [13]. In that implementation, the state variable $z(t)$ in Eq. (1), which is the current through the inductor, can be determined by measuring a voltage in a node inside that network. The implication is that the $z(t)$ variable can be changed by another voltage variable $v(t)$, using the expression

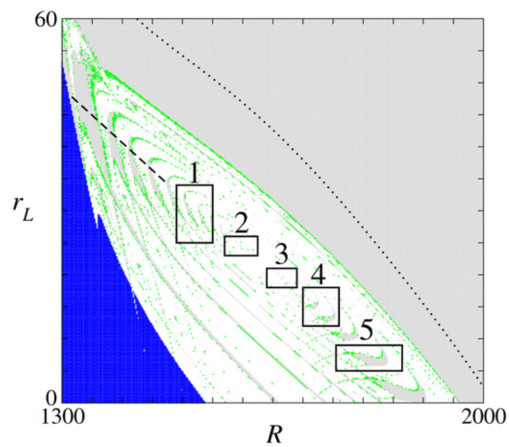

Fig. 1. (Color online.) Parameter-space diagram associating colors to different λ ranges. Grey color for periodic orbits (left of dotted line) and for fixed points (right of dotted line), green color for the transition of periodic to chaotic orbits, white color for chaotic orbits, and blue color for divergence region. Regions inside the boxes appear amplified in Figs. 2 and 5. Here and hereafter, R and r_L are in resistance units, Ω.

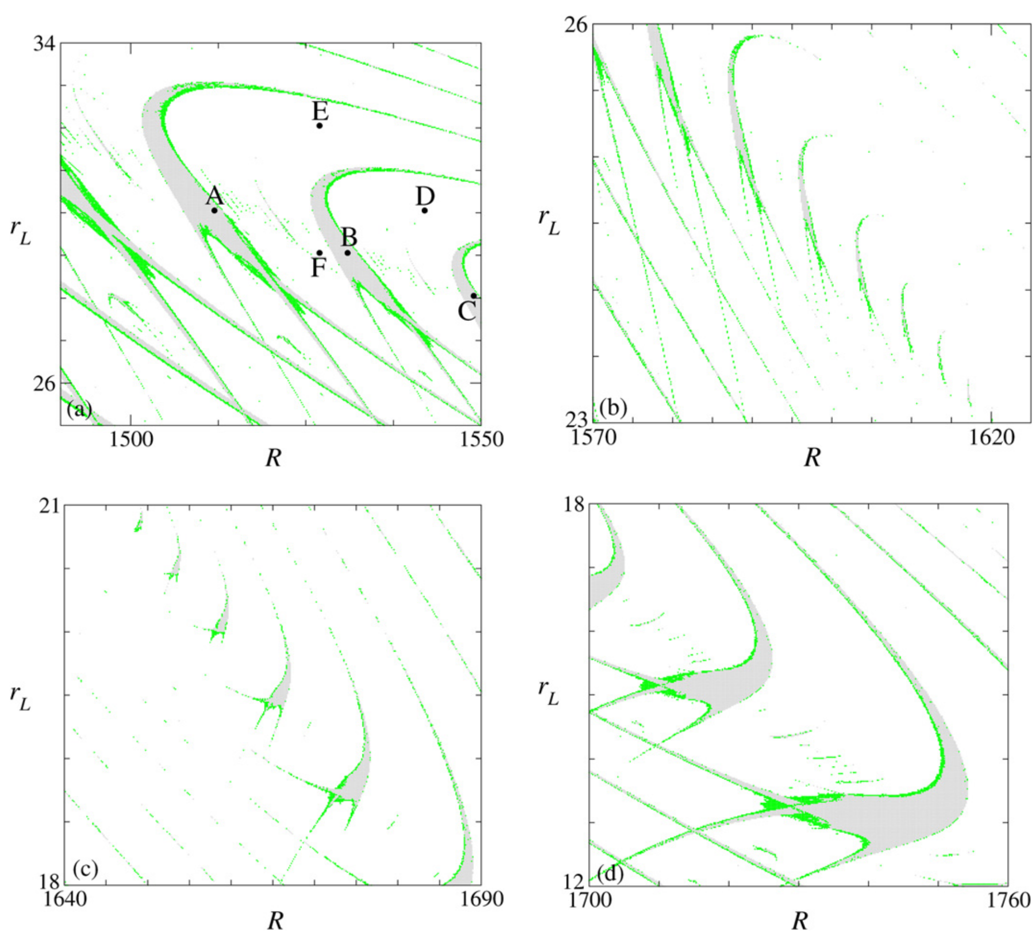

Fig. 2. (Color online.) Magnification of the four boxes (1, 2, 3, and 4) in Fig. 1, showing details of the parameter-space. Color scheme is the same used as in Fig. 1. The points A, B, C, D, E, and F in (a) are the attractors in Fig. 4.