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Adaptive synchronization between different hyperchaotic systems with fully uncertain parameters

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ABSTRACT

Adaptive method is an effective way to synchronize different hyperchaotic systems. This work investigates the chaos synchronization between different hyperchaotic systems with fully unknown parameters, i.e., the synchronizations between Lorenz–Stenflo (LS) system and a novel dynamical system named CYQY system, and between LS system and hyperchaotic Chen system. Based on the Lyapunov stability theory, two new adaptive controllers with corresponding parameter update laws are designed such that the different hyperchaotic systems can be synchronized asymptotically. Numerical simulations are presented to demonstrate the effectiveness of the proposed adaptive controllers.

1. Introduction

Since Rössler firstly introduced a hyperchaotic dynamical system [1], many hyperchaotic systems have been proposed and studied in the last decade, for example, the hyperchaotic Chen system [2,3], a new hyperchaotic Lorenz system [4], the hyperchaotic Lü system [5], the hyperchaotic LS system and the hyperchaotic Qi system [6], just to name a few. Hyperchaotic system has more than one positive Lyapunov exponent which generates more complex dynamics than the low-dimensional chaotic system. Therefore, hyperchaotic system has much wider application than the low-dimensional chaotic system. For example, the adoption of hyperchaotic system has been proposed for secure communication and the presence of more than one positive Lyapunov exponent clearly improves the security of the communication scheme [7].

Until now, a variety of approaches have been proposed for the synchronizations of low-dimensional chaotic systems such as the linear and nonlinear feedback synchronization method [8–11], adaptive synchronization method [12–16], time-delay feedback method [17], backstepping design method [18,19] and sliding mode method [20], impulsive synchronization method [21,22], etc. Fortunately, many existing methods of synchronizing low-dimensional chaotic systems can be generalized to synchronize hyperchaotic systems such as the adaptive controller method [23–26], projective method [27,28], feedback method [29,30], backstepping nonlinear method [31], lag synchronization method [32], impulsive method [33,34], function cascade method [35] and so on. Among these synchronization methods, the adaptive control is proved to be an effective one to achieve the synchronization between different hyperchaotic systems with uncertain parameters.

Recently, the synchronizations of hyperchaotic systems by adaptive controllers have attracted much attention from nonlinear area. Some researchers investigated the synchronizations between identical hyperchaotic systems, for example, the synchronization of hyperchaotic Rössler systems [36,37], hyperchaotic Lü systems [38], hyperchaotic Chen systems [39], hyperchaotic Liu system [40], etc. At the same time, the synchronizations between different hyperchaotic systems have also been considered, such as the synchronizations between generalized Henon–Heiles system and hyperchaotic Chen system [27], between hyperchaotic Chen system and second-harmonic generation (SHG) system [41], between hyperchaotic Lorenz system and hyperchaotic Liu system [42], between hyperchaotic Chen system and a new hyperchaotic system [42], between hyperchaotic Lorenz system and hyperchaotic Lü system [43] and others [44,45]. Differs from these studies, the reduced-order adaptive synchronization between generalized Lorenz system

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(fourth order) and Lü system (third order), between Rössler hyperchaotic system (fourth order) and Rössler system (third order) have been studied in Ref. [46]. As we all know, the synchronization between different hyperchaotic systems is very important in engineering application because they have different complex dynamics behaviors. Contributing to this line of literature, this work focuses on the synchronization between different hyperchaotic systems. However, differs from the existing works, this work aims at the chaos synchronization between hyperchaotic LS system and hyperchaotic CYQY system, between LS system and hyperchaotic Chen system with fully uncertain parameters. Based on the Lyapunov stability theory, two new adaptive synchronization controllers with corresponding parameter update laws are designed such that the LS system and the hyperchaotic CYQY system, the LS system and the hyperchaotic Chen system can be synchronized globally and effectively.

The rest of the Letter is organized as follows. Section 2 briefly describes the LS system, hyperchaotic CYQY system and hyperchaotic Chen system, respectively. Section 3 presents a novel adaptive synchronization controller with corresponding parameter update laws to synchronize LS system and hyperchaotic CYQY system whose effectiveness are verified by some numerical simulations in Section 4. Section 5 proposes another adaptive controller for the synchronization between LS system and hyperchaotic Chen system and Section 6 gives some numerical simulations. A conclusion is given in the end.

2. Systems description

Hyperchaotic Lorenz–Stenflo (LS) system is described as [47]

$$\begin{cases} \dot{x} = \alpha(y - x) + \gamma w, \\ \dot{y} = x(r - z) - y, \\ \dot{z} = xy - \beta z, \\ \dot{w} = -x - \alpha w, \end{cases} \tag{1}$$

which was formulated by Stenflo from a low-frequency short-wavelength gravity wave equation. In Eq. (1), x, y, z and w are state variables and $r(>0)$, $\alpha(>0)$, $\gamma(>0)$, $\beta(>0)$ are the related parameters named Rayleigh number, Prandtl number, rotation number and geometric parameter, respectively. System (1) is generated from the originally three-dimensional Lorenz chaotic system by introducing a new control parameter γ and a state variable w which describes the flow rotation. When $\alpha = 1.0$, $\beta = 0.7$, $\gamma = 1.5$ and $r = 26.0$, system (1) exhibits hyperchaotic behaviors (Fig. 1(a)). Some dynamical behaviors have been studied for LS system including the familiar period-doubling route to chaos [48], an extension of the chaotic scenario [49], the phase synchronization and adaptive synchronization between two LS systems [50], the reduced-order adaptive synchronization between LS system and Lü system [46], the active synchronization [6] and backstepping synchronization between LS system and Qi system [51].

Recently, based on the famous Lorenz system, Chen et al. introduced a novel hyperchaotic system, named CYQY system which is given by the following equations [52]

$$\begin{cases} \dot{x} = l(y - x) + kyz, \\ \dot{y} = nx - jxz + y + w, \\ \dot{z} = xy - mz, \\ \dot{w} = -\lambda y, \end{cases} \tag{2}$$

where l, m, n, j, k, λ are constant parameters and x, y, z, w are state variables. This system can generate complex dynamics behaviors including chaos, Hopf bifurcation, period-doubling bifurcation, sink and so on. When $l = 35$, $m = 4.9$, $n = 25$, $j = 5$, $k = 35$, $\lambda = 100$, system (2) can exhibit a complex hyperchaotic attractor (Fig. 1(b)). Moreover, system (2) is symmetric with respect to the z-axis and is dissipative when $a + b - 1 > 0$. In addition, this system has only one unstable zero equilibrium, which is a saddle point [52].

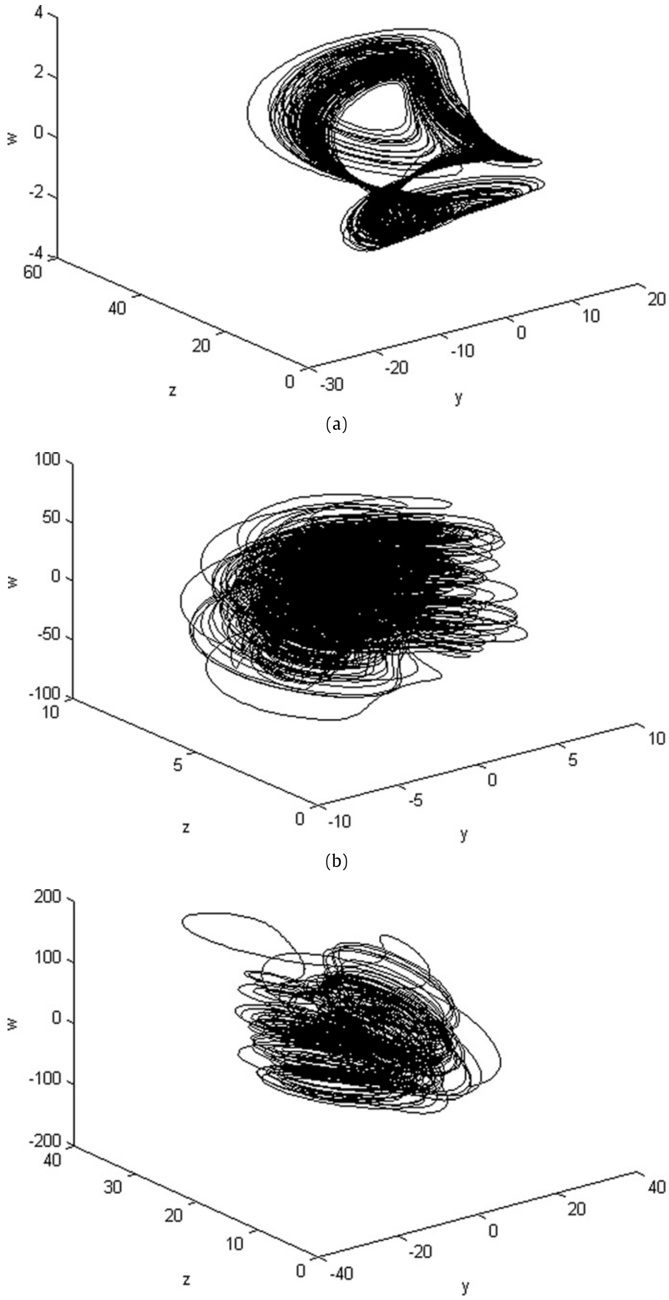

Fig. 1. Hyperchaotic attractors: (a) hyperchaotic LS system in (y, z, w) space; (b) Hyperchaotic CYQY system in (y, z, w) space; (c) Hyperchaotic Chen system in (y, z, w) space.

Hyperchaotic Chen system is described as [4,39]

$$\begin{cases} \dot{x} = a(y - x) + w, \\ \dot{y} = dx - xz + cy, \\ \dot{z} = xy - bz, \\ \dot{w} = yz + hw, \end{cases} \tag{3}$$

where x, y, z and w are state variables and a, b, c, d and h are real constant parameters. When $a = 35$, $b = 3$, $c = 12$, $d = 7$, $0 \leqslant h \leqslant 0.085$, system (3) is chaotic; when $a = 35$, $b = 3$, $c = 12$, $d = 7$, $0.085 \leqslant h \leqslant 0.798$, system (3) exhibits hyperchaotic behavior (Fig. 1(c)); when $a = 35$, $b = 3$, $c = 12$, $d = 7$, $0.798 \leqslant h \leqslant 0.9$, system (3) is periodic. Some dynamics behavior of hyperchaotic Chen system have been investigated including the control [3], the