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A novel delay-dependent criterion for delayed neural networks of neutral type

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ABSTRACT

This Letter considers a robust stability analysis method for delayed neural networks of neutral type. By constructing a new Lyapunov functional, a novel delay-dependent criterion for the stability is derived in terms of LMIs (linear matrix inequalities). A less conservative stability criterion is derived by using nonlinear properties of the activation function of the neural networks. Two numerical examples are illustrated to show the effectiveness of the proposed method.

1. Introduction

The application of neural networks is found in various fields such as signal processing, content-addressable memory, and optimization [1–3]. Also, dynamics of neural networks have been intensively studied because of the successful hardware implementation and their wide applications [4–6]. However, in hardware implementation, time delays are frequently encountered because of the finite propagation speed of signal or the finite switching speed of amplifier in neural circuits. Their existence may cause instability and oscillation of neural networks. Therefore, considerable efforts have been done to the global asymptotic stability of delayed neural networks [3–13]. The stability analysis of neural networks with time-delay have been extensively studied to different versions of the stability problem [14–17].

Recently, the stability analysis of neural networks with neutral-type has been investigated due to the complicated dynamic properties of the system that contain the information about the derivative of the past state and the dynamics for complex neural reactions [10]. Practically, the phenomena on neutral delay often appears in the study of automatic control, population dynamics [18], partial element equivalent circuits [19] and transmission lines [20] in electrical engineering, controlled constrained manipulators in mechanical engineering [21] and so on. In this field of neural networks, an important index for checking the conservatism of stability criteria is to increase the feasible region of stability criteria or to get the maximum allowable bounds of time delays for guaranteeing the stability of the networks.

In this Letter, a new delay-dependent stability criterion for neural networks with neutral-type time-varying delays is proposed. In order to derive a less conservative stability criterion, a new Lyapunov function is proposed using properties of the nonlinearity of the activation function. In the existing results, the activation function of neural networks is assumed to be nondecreasing, bounded and globally Lipschitz. Generally, the saturation or tangent hyperbolic function is frequently used as an activation function of the physical cellular neural networks [2]. Thus, a novel delay-dependent criterion for the stability of the system is derived in terms of LMIs using the transformed system with the deadzone nonlinearity. Since the criterion is derived from the sector and slope property of the deadzone nonlinearities, our result can be less conservative than the existing ones. Finally, the effectiveness of the proposed method is illustrated by two numerical examples.

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Throughout the Letter, $\mathbb{R}^n$ denotes n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. $X<0$ means that X is a real symmetric negative definitive matrix. I denotes the identity matrix with appropriate dimensions. $\mathrm{diag}\{\cdots\}$ denotes the diagonal matrix.

2. Problem statement and preliminaries

Consider the following the delayed neural networks of neutral type

$$\dot{x}(t)=-Ax(t)+W_1\bar{f}\big(x(t)\big)+W_2\bar{f}\big(x\big(t-h(t)\big)\big)+V\dot{x}\big(t-\tau(t)\big)+b, \tag{1}$$

where n denotes the number of neurons in a neural network, $x(t)=[x_1(t),\dots,x_n(t)]^T\in\mathbb{R}^n$ is the neuron state vector, $\bar{f}(x(t))=[\bar{f}_1(x_1(t)),\dots,\bar{f}_n(x_n(t))]^T\in\mathbb{R}^n$ and $\bar{f}(x(t-h(t)))=[\bar{f}_1(x_1(t-h(t))),\dots,\bar{f}_n(x_n(t-h(t)))]^T\in\mathbb{R}^n$ is the activation functions, $b=[b_1,\dots,b_n]^T$ is the external bias at time t, $A=\mathrm{diag}(a_i)$ is a positive diagonal matrix, $W_1=(w_{ij}^1)_{n\times n}$, $W_2=(w_{ij}^2)_{n\times n}$ and $V=(v_{ij})_{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons, $h(t)>0$ and $\tau(t)>0$ correspond to finite speed of axonal signal transmission delay satisfying the following:

$$0\leqslant h(t)\leqslant h_M,\qquad 0\leqslant\tau(t)\leqslant\tau_M,\qquad \dot{h}(t)\leqslant h_D,\qquad \dot{\tau}(t)\leqslant\tau_D<1.$$

Throughout the Letter, it is assumed that the neurons activation functions $\bar{f}$ is saturation function defined as

$$\mathrm{sat}\big(x_i(t)\big)=\begin{cases}x_i(t), & |x_i(t)|\leqslant x_i^{\max},\\ \mathrm{sign}(x_i(t)), & |x_i(t)|>x_i^{\max},\end{cases} \tag{2}$$

where $x_i^{\max}$ is a positive saturation level. Then, one can easily prove that there exists at least on equilibrium point x^* for the systems (1) using the well-known Brouwer's fixed-point theorem [3]. Therefore, the system (1) can be transformed by shifting the equilibrium point to the origin $y(t)=x(t)-x^*$.

$$\dot{y}(t)=-Ay(t)+W_1f\big(y(t)\big)+W_2f\big(y\big(t-h(t)\big)\big)+V\dot{y}\big(t-\tau(t)\big) \tag{3}$$

where $y(t)$ is the state vector of the transformed system, $f(x)=[f_1(y),\dots,f_n(y)]^T$, $f_j(y_j(t))=\bar{f}_j(y_j(t)+x_j^*)-\bar{f}_j(x_j^*)$ and f_j satisfy the saturation function property as Eq. (2).

Now, define the following parameters k_i, K and deadzone nonlinearity $\phi(y(t))$:

$$k_i=\frac{1}{x_i^{\max}}, \tag{4}$$

$$K=\mathrm{diag}\{k_1,k_2,\dots,k_m\}, \tag{5}$$

$$\phi\big(y(t)\big)=Ky(t)-f\big(y(t)\big). \tag{6}$$

Then, the function $\phi_i(y(t))$ belongs to the sector $[0,k_i]$ with the slope constraints, i.e.,

$$0\leqslant\frac{\phi_i(y_i(t))}{y_i(t)}\leqslant k_i, \tag{7}$$

$$\frac{d\phi_i(y_i(t))}{dy_i(t)}=\begin{cases}0, & |y_i(t)|\leqslant y_i^{\max},\\ k_i, & |y_i(t)|>y_i^{\max},\end{cases} \tag{8}$$

where k_i is a sector and slope bound of ϕ_i and $y_i^{\max}$ is a positive deadzone level that is the same value of $x_i^{\max}$.

From the slope conditions Eq. (8), we can find the following properties

$$\big(k_i-\phi_i'\big(y_i(t)\big)\big)\phi_i\big(y_i(t)\big)=0, \tag{9}$$

$$\big(k_i-\phi_i'\big(y_i(t)\big)\big)\phi_i'\big(y_i(t)\big)=0 \tag{10}$$

where $\phi_i'(y_i(t))=d\phi_i(y_i(t))/dy_i(t)$.

Note that $d\phi_i(y_i(t))/dy_i(t)$ may not exist where $y_i(t)=\pm y_i^{\max}$.

Using the nonlinear function (6), the transformed system (3) can be rewritten as

$$\dot{y}(t)=(-A+W_1K)y(t)+W_2Ky\big(t-h(t)\big)-W_1\phi\big(y(t)\big)-W_2\phi\big(y\big(t-h(t)\big)\big)+V\dot{y}\big(t-\tau(t)\big). \tag{11}$$

The purpose of this Letter is to find the maximum allowable delay bound such that the dynamics, Eq. (11), of the system is globally stable.

The following lemma is useful for deriving the stability criterion.

Lemma 1. *(See [22].) For any constant symmetric matrix $M\in\mathcal{R}^{n\times n}$, a positive scalar γ, and a vector function $x:[0,\gamma]\to\mathcal{R}^n$ such that the integrations concerned are well defined, then*

$$\left(\int_0^\gamma x(s)\,ds\right)^T M\left(\int_0^\gamma x(s)\,ds\right)\leqslant\gamma\int_0^\gamma x^T(s)Mx(s)\,ds.$$