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Effects of dust temperature and trapped ions on the formation of dust-acoustic solitary waves

H. Alinejad [a,b,*]

[a] Department of Basic Science, Babol University of Technology, Babol 47148-71167, Iran
[b] Research Institute for Astronomy and Astrophysics of Maragha, P.O. Box 55134-441, Maragha, Iran

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ABSTRACT

The effects of dust temperature and trapped ions are incorporated in the study of dust-acoustic solitary waves. An energy integral equation involving the Sagdeev potential is derived, and the basic properties of large amplitude solitary structures are investigated. It is shown that the effects of dust temperature, resonant ions and equilibrium free electron density significantly change the regions of the existence of large amplitude solitary waves. Expanding the Sagdeev potential to include higher-order nonlinearities of electric potential, an exact steady state solution is also obtained which confirms the possibility of dust-acoustic soliton in the small amplitude limit. Furthermore, two asymptotic cases of the stationary solution are found which are related to the contribution of trapped ions.

1. Introduction

There has been a great deal of interest in numerous collective processes in dusty plasmas, i.e., plasmas with extremely massive and negatively charged dust grains. Such plasmas occur in laboratory, astrophysical and space environments, such as cometary tails, planetary rings, interstellar medium, the earth's environment, etc. [1–4]. Due to the presence of the charged dust grains in plasmas, different types of collective processes exist and very rich wave modes can be excited in dusty plasmas [5–8]. One of these is the low frequency dust-acoustic mode [9,10] in an unmagnetized dusty plasma whose constituents are an inertial charge dust fluid and Boltzmann distributed electrons and ions. Thus, in the dust-acoustic waves the dust particle mass provides the inertia and the thermal pressure from the electrons and ions give rise to the restoring force. Rao et al. [9] were the first to report theoretically the existence of dust-acoustic solitary waves by using the reductive perturbation method which is only valid for small but finite amplitude limit. The predictions of Rao et al. [9] were conclusively verified by the laboratory experiment of Barkan [10].

On the other hand, numerical simulation studies [11] on linear and nonlinear dust-acoustic waves exhibit a significant amount of ion trapping in the wave potential. Clearly, there is a departure from the Boltzmann ion distribution and one encounters the vortex-like ion distribution in the phase space. It is well known that such ion behavior drastically modifies the conditions for the existence of nonlinear structures such as solitons and shocks, which are not observed in dusty plasma with isothermal ions [12–17]. Most of the studies discussed up to now are restricted to theoretical investigations on soliton dynamics in a dusty plasma with cold dust grains in the frame work of the modified KdV equation using the reductive perturbation method. However the perturbation method is mainly valid for the small amplitude solitary waves, Sagdeev potential approach [18] is a powerful tool for studying large amplitude solitary structures. Adopting this approach, Roychoudhury et al. [19] investigated the large amplitude solitary waves in the finite temperature dusty plasma with isothermal ions. They showed that the dust temperature can restrict the region of existence of dust-acoustic solitary waves. Akhtar et al. [20] studied the nonlinear dynamics of dust-acoustic waves in unmagnetized multicomponent plasmas with hot and cold dust species. They investigated the effects of different masses, charges and concentration ratios of the dust particles on the shape of solitary structures. Mahmood et al. [21] examined the influence of dust temperature on nonlinear dust-acoustic waves in a magnetized plasma. They also showed that the dust thermal energy reduces the wave amplitude in a magnetized plasma, which has the same behavior as in an unmagnetized case.

A little attention has been paid to study the effects of dust temperature and deviations from isothermality of ions on the propagation of large amplitude solitary waves in dusty plasma. For example, Mendoza et al. [22] investigated the effects of dust fluid temperature and fast or non-thermal ions on arbitrary ampli-

* Address for correspondence: Department of Basic Science, Babol University of Technology, Babol 47148-71167, Iran. Tel.: +98 111 3234203; fax: +98 111 3234201.
E-mail address: alinejad@nit.ac.ir.

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tude dust-acoustic solitary structures. They showed that a suitable distribution of non-thermal ions in a dusty plasma system may change the nature of the dust-acoustic solitary waves and may support the coexistence of large amplitude compressive and rarefactive solitary waves. Also, Das et al. [23] investigated arbitrary amplitude dust-acoustic solitary waves and double layers in non-thermal plasma with the effect of dust temperature. They showed that the amplitude of the positive potential double layer decreases with increasing the dust temperature.

To our knowledge, large amplitude dust-acoustic solitary waves including the effects of dust temperature and trapped ions has not been investigated before. Hence the main motivation of the present work is to study the effects of dust temperature and higher-order nonlinearities on the dust-acoustic solitary waves in a dusty plasma with ions following a vortex-like distribution. The trapped ions are included in this model as a result of nonlinear resonant interaction of the localized electrostatic wave potential with ions during its evolution. The electron inertia is neglected and Boltzmann distribution for the electron density is assumed which implies isothermality. To study the regions of existence of arbitrary amplitude solitary waves an energy integral equation involving a Sagdeev potential is derived. Moreover, to confirm the possibility of the dust-acoustic soliton, we investigate higher-order nonlinear and dispersive effects of the electric potential in the expansion of the Sagdeev potential. We also investigate two asymptotic cases of the stationary solution which are related to the contribution of the resonant ions.

2. Basic equations and localized waves

We consider a collisionless, unmagnetized three component dusty plasma consisting of extremely massive warm dust grains, Boltzmann distributed electrons and hot ions obeying a vortex-like distribution. Although the size and the charge of the dust grains varies from one grain to another, we assume for simplicity that all grains have the same negative charge, $q_d = -Z_d e$, where Z_d is the number of charges residing on the surface of the dust grains. At equilibrium, we have $n_{i0} = Z_d n_{d0} + n_{e0}$, where n_{i0}, n_{d0} and n_{e0} are the unperturbed ion, dust and electron number densities, respectively. The dynamics of the nonlinear dust-acoustic waves in a three component dusty plasma system with a constant dust grain charge is thus governed by [9]

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(NU) = 0, \tag{1}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \frac{\sigma_d}{N}\frac{\partial P}{\partial x} = \frac{\partial \psi}{\partial x}, \tag{2}$$

$$\frac{\partial P}{\partial t} + U\frac{\partial P}{\partial x} + 3P\frac{\partial U}{\partial x} = 0, \tag{3}$$

$$\frac{\partial^2 \psi}{\partial x^2} = N - \mu_1 N_i + \mu_0 e^{\sigma\psi}, \tag{4}$$

where N is the dust particle number density normalized to its equilibrium value n_{d0}, N_i is the ion number density normalized to n_{i0}, U is the dust fluid velocity normalized to the dust-acoustic speed $C_d = (Z_d T_i/m_d)^{1/2}$, and ψ is the electrostatic wave potential normalized to T_i/e, and P is the dust pressure normalized to $n_{d0}T_d$, where T_d is the dust fluid temperature and m_d is the mass of negatively charged dust particulates. $\sigma = T_i/T_e$, with T_i (T_e) being the ion (electron) temperature, $\mu_0 = \beta/(1-\beta)$ and $\mu_1 = 1/(1-\beta)$, where $\beta = n_{e0}/n_{i0}$. The time and space variables are given in the units of the dust plasma period $\omega_{pd}^{-1} = (m_d/4\pi Z_d^2 n_{do} e^2)^{1/2}$ and the Debye length $\lambda_{Dd} = (T_i/4\pi Z_d n_{do} e^2)^{1/2}$, respectively.

The model used here is based on the assumption of neglecting some of dust grains which might be trapped due to the finite temperature in the wave potential. The electron inertia is neglected. This expression is obtained by the consideration that a thermal electron moves with a speed much higher than the ion thermal speed. In this case, ions could interact with the wave potential during its evolution, and therefore can be trapped in the wave potential. Then, to model an ion distribution with trapped particles, we employ a vortex-like ion distribution function of Schamel [24,25], which solves the ion Vlasov equation. In such environment, the ion number density can be expressed as

$$N_i = I(-\psi) + \frac{e^{-\alpha\psi}}{\sqrt{|\alpha|}} erf(\sqrt{-\alpha\psi}), \tag{5}$$

where

$$I(-\psi) = [1 - erf(\sqrt{-\psi})]e^{-\psi},$$

$$erf(\sqrt{-\alpha\psi}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{-\alpha\psi}} e^{-t^2}\, dt.$$

Here $\alpha = T_i/T_{it}$ (ratio of the free ion temperature to trapped ion temperature) is the trapping parameter describing the temperature of the trapped ions. Positive values of α, which we are interested in here, lead to a vortex-like distribution for electrons. Note that if we neglect the resonant effects, N_i reduces to the Maxwellian ion distribution.

To obtain stationary localized solution from the basic equations, we make all the dependent variables depend only on a single variable $\xi = x - Mt$, where ξ is normalized to λ_{Dd} and M is the Mach number of the solitary waves with respect to the dust-acoustic wave frame for the system. Then, using the steady state condition ($\partial/\partial t = 0$) and the appropriate boundary conditions for localized perturbations, namely, $N \to 1$, $U \to 0$, $\psi \to 0$, $P \to 1$ and $d\psi/d\xi \to 0$ at $\xi \to \pm\infty$, in Eqs. (1)–(3), one can express the dust particle number density N as

$$N = \sqrt{2}M\Big[M^2 + 3\sigma_d + 2\psi + \sqrt{(M^2 + 3\sigma_d + 2\psi)^2 - 12\sigma_d M^2}\,\Big]^{-\frac{1}{2}}. \tag{6}$$

Now using (6), the qualitative nature of the solutions of Eq. (4) is most easily seen by introducing the Sagdeev potential [18]. Therefore, Poisson's equation (4) reduces to the form

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 + V(\psi) = 0, \tag{7}$$

where the Sagdeev potential $V(\psi)$ is defined as

$$\begin{aligned} V(\psi) = {} & \frac{\mu_0}{\sigma}(1 - e^{\sigma\psi}) + \mu_1[1 - I(-\psi)] - \frac{2\mu_1}{\sqrt{\pi}}\left(1 - \frac{1}{\alpha}\right)\sqrt{-\psi} \\ & - \frac{\mu_1 e^{-\alpha\psi}}{\sqrt{\alpha^3}} erf(\sqrt{-\alpha\psi}) \\ & - \frac{\sqrt{2}M}{3\gamma^{3/2}} e^{\frac{1}{2}\cosh^{-1}(\chi/\gamma)}\Big[\gamma^2 + \chi\big(\chi - \sqrt{\chi^2 - \gamma^2}\big)\Big] \\ & + \frac{\sqrt{2}M}{3\gamma^{3/2}} e^{\frac{1}{2}\cosh^{-1}(\eta/\gamma)}\Big[\gamma^2 + \eta\big(\eta - \sqrt{\eta^2 - \gamma^2}\big)\Big], \end{aligned} \tag{8}$$

with

$$\chi = M^2 + 3\sigma_d + 2\psi, \tag{9}$$

$$\gamma = 2M\sqrt{3\sigma_d}, \tag{10}$$

$$\eta = M^2 + 3\sigma_d. \tag{11}$$

We note that Eq. (7) can be regarded as an "energy integral" of an oscillating particle of unit mass, with pseudo-speed $d\psi/d\xi$,