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The single vortex in the FFLO state

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ABSTRACT

In this Letter, we study the generalized Ginzburg–Landau (GL) functional near the tricritical temperature, and obtain the vortex solution of the FFLO state. Furthermore, we investigate the structure of the vortex and find that the vortices shrink when the Zeeman effect is weaken or temperature is lowered.

1. Introduction

In 1964 Fulde and Ferrell (FF) [1], Larkin and Ovchinnikov (LO) [2] predicted an exotic superconducting state with a spatially modulated order parameter at low temperatures and high fields when the paramagnetic depairing is significant due to the Zeeman effect under a magnetic field. The FFLO state was observed in the heavy fermion superconductor CeCoIn5 in the high magnetic field region of the superconducting phase diagram [3], which arouse tremendous interest both in experiment and theory. These states have also been argued to be important to understand ultracold atomic Fermi gases [4], color superconductivity in high density quark matter [5] and isospin asymmetric nuclear matter with proton–neutron paring [6,7]. In these states vortices play an important role (in CeCoIn5 the FFLO state appears deep within a vortex state [3] and ultracold atomic Fermi gases can be rotated to create vortices within a FFLO state [8]). Thus one central issue of these states is to understand properties of the vortices.

Previous studies indicate that the order parameter in the FFLO vortex state is, for example, $\Psi(\vec{R}) = \phi_n(\vec{r})\cos(qz)$, where $\hat{z}$ is along the direction of the magnetic filed, $\hat{z}\cdot\vec{r} = 0$, $\phi_n(\vec{r})$ describes the structure of vortex lattice [9–13]. This solution has spatial nodes along the z-axis and the vortex lines parallel to the z-axis where the un-paired component is laid [9]. In this Letter, we study a single vortex of the special superconductor state and get the numerical solution. With the solution we discuss some physical properties of the state. The Letter is organized as follows. In Section 2 we give the formulism of FFLO states. In Section 3 we derive the equation of motion and discuss the asymptotic behavior of the equation and give numerical results of vortex in LO state. The discussion of the FF state and numerical result is presented in Section 4. We finish in Section 5 with a discussion of our results and present our conclusions.

2. Generalized GL theory and the equation of motion

We start with the generalized GL theory pioneered by Buzdin and Kachkachi to describe the FFLO state [10,11,14,15]. The free energy density is

$$\mathcal{F} = \alpha|\Psi|^2 + \beta|\partial\Psi|^2 + \gamma|\Psi|^4 + \delta\left|\partial^2\Psi\right|^2 + \mu|\Psi|^2|\partial\Psi|^2 + \frac{\mu}{8}\left[\left(\Psi^\dagger\right)^2(\partial\Psi)^2 + (\Psi)^2\left(\partial\Psi^\dagger\right)^2\right] + \nu|\Psi|^6 \qquad (1)$$

where the coefficients are

$$\alpha = \pi\mathcal{N}(0)\left(\mathcal{K}_1^0 - \mathcal{K}_1\right), \qquad \gamma = \frac{\pi\mathcal{N}(0)\mathcal{K}_1}{4},$$

$$\nu = -\frac{\pi\mathcal{N}(0)\mathcal{K}_5}{8}, \qquad \beta = \frac{\pi\mathcal{N}(0)\mathbb{V}_F^2\mathcal{K}_3}{4},$$

$$\delta = \frac{\pi\mathcal{N}(0)\mathbb{V}_F^4\mathcal{K}_5}{80}, \qquad \mu = \frac{\pi\mathcal{N}(0)\mathbb{V}_F^2\mathcal{K}_5}{6}.$$

$\mathbb{V}_F$ is Fermi velocity, $\mathcal{N}(0)$ is electron density of state, and $\mathcal{K}_n = 2\mathcal{T}Re[\sum_{\nu=0}^{\infty}\frac{1}{(\omega_\nu - \imath\mathcal{I})^n}]$, $n \geqslant 1$; $\mathcal{I}$ is the "exchanged filed or the magnetic field"; $\omega_\nu = (2\nu+1)\pi\mathcal{T}$ are Matsubara's frequencies at temperature $\mathcal{T}$. The expression of $\mathcal{F}$ describes the dispersion of free energy between the FFLO state and normal state. Thus $\mathcal{F}$ is always

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negative for FFLO state [16]. In contrast to the standard GL theory, terms with a second order derivative of the order parameter, $(\Psi'')^2$ as well as terms $\Psi^2(\Psi')^2$ are added. When the coefficient β changes its sign, the non-uniform state appears. In the standard GL functional, β is positive, but it occurs to be a function of the field which act as Zeeman effect, and goes to zero at $(\mathcal{T}^*, \mathcal{I}(\mathcal{T}^*))$ being negative at $\mathcal{T} < \mathcal{T}^*$. Negative β means that the modulated state has lower energy compared with the non-uniform one [16]. We normalize the coefficients in the following way [16].

$$\Psi = \tilde{\Psi}\sqrt{\frac{\mathcal{K}_3}{\mathcal{K}_5}}, \qquad \mathcal{X} = \frac{\tilde{\mathcal{X}}}{\sqrt{\frac{10\mathcal{K}_3}{3\mathcal{K}_5 \mathbb{V}_F^2}}},$$

$$\tilde{\mathcal{F}} = \frac{\mathcal{F}}{\frac{-\mathcal{K}_5}{8}\left(\frac{\mathcal{K}_3}{\mathcal{K}_5}\right)^3}, \qquad \tilde{\alpha} = \frac{\alpha}{\frac{-\mathcal{K}_5}{8}\left(\frac{\mathcal{K}_3}{\mathcal{K}_5}\right)^2}.$$

Then the free energy per unit volume is

$$\tilde{\mathcal{F}} = \frac{1}{\tilde{V}}\int d^3\tilde{\mathcal{X}}\left\{\tilde{\alpha}|\tilde{\Psi}|^2 - \frac{20}{9}|\partial\tilde{\Psi}|^2 - 2|\tilde{\Psi}|^4 + \frac{10}{9}|\partial^2\tilde{\Psi}|^2 + \frac{40}{9}|\tilde{\Psi}|^2|\partial\tilde{\Psi}|^2 + \frac{5}{9}\left[(\tilde{\Psi}^\dagger)^2(\partial\tilde{\Psi})^2 + (\tilde{\Psi})^2(\partial\tilde{\Psi}^\dagger)^2\right] + |\tilde{\Psi}|^6\right\}. \tag{2}$$

Firstly we choose the simplest exponential order parameter $\tilde{\Psi} = \psi_0 e^{\imath qz}$ for FF state, the free energy reads $\tilde{\mathcal{F}} = \tilde{\alpha}\psi_0^2 - \frac{20}{9}q^2\psi_0^2 + \frac{10}{9}q^4\psi_0^2 - 2\psi_0^4 + \frac{10}{3}q^2\psi_0^4 + \psi_0^6$. It is easy to find three q for the minimum of $\tilde{\mathcal{F}}$, namely 0, $\pm[\frac{5}{9} + (\frac{\tilde{\alpha}}{2} - \frac{29}{81})^{\frac{1}{2}}]^{\frac{1}{2}}$. The first one is connected to the uniform state which is trivial case. And for FF state the last two values are equivalent which are called "proper" value. Without loss of generality, we choose the positive q for the "proper" value in our discussion. Similarly, for LO state $\tilde{\Psi} = \psi_0\cos(qz)$, the proper q is $[0.879 - (0.772 - 0.682\tilde{\alpha})^{\frac{1}{2}}]^{\frac{1}{2}}$.

Next we consider a single vortex in a superconductor along the z-axis with cylindrical symmetry. The modulation vector of order parameter is along the z-axis too. Thus the order parameter could be written as: $\tilde{\Psi} = \varphi(r)e^{\imath m\theta}\cos(qz)$ (for LO state) and $\tilde{\Psi} = \varphi(r)e^{\imath m\theta}e^{\imath qz}$ (for FF state). m is the winding number of the vortex, and $|\varphi(r)|^2$ describes the radial distribution of the density. Without loss of generality, we consider $\varphi(r)$ as positive real function.

3. Vortex in the LO state

In this section we focus on the vortex in LO state. We derive the equation of motion of LO state. Then we analyze the solution and give the numerical result. Following that we consider the structure of the vortex of LO state.

3.1. The equation of motion in LO state

Substituting $\tilde{\Psi} = \varphi(r)e^{\imath m\theta}\cos(qz)$ into (2) and integrating z and θ we find that (for LO state)

$$\begin{aligned}\tilde{\mathcal{F}} = \frac{2}{r_0^2}\int r\,dr\Bigg[&\frac{\tilde{\alpha}\varphi(r)^2}{2} - \frac{3\varphi(r)^4}{4} + \frac{5\varphi(r)^6}{16} - \frac{10q^2\varphi(r)^2}{9} \\ &+ \frac{5}{9}q^4\varphi(r)^2 + \frac{25}{36}q^2\varphi(r)^4 + \frac{10m^2q^2\varphi(r)^2}{9r^2} \\ &+ \frac{5m^4\varphi(r)^2}{9r^4} - \frac{10m^2\varphi(r)^2}{9r^2} + \frac{5m^2\varphi(r)^4}{4r^2} \\ &- \frac{10m^2\varphi(r)\varphi'(r)}{9r^3} - \frac{10}{9}\varphi'(r)^2 - \frac{10q^2\varphi(r)\varphi'(r)}{9r} \\ &+ \frac{5\varphi'(r)^2}{9r^2} + \frac{25}{12}\varphi(r)^2\varphi'(r)^2 - \frac{10}{9}q^2\varphi(r)\varphi''(r) \\ &- \frac{10m^2\varphi(r)\varphi''(r)}{9r^2} + \frac{10\varphi'(r)\varphi''(r)}{9r} + \frac{5}{9}\varphi''(r)^2\Bigg].\end{aligned} \tag{3}$$

Here we consider an axially symmetric system with a radius r_0 and one period $\frac{2\pi}{q}$ in the z-axis and the radial coordinate r has been normalized. Thus we can obtain the equation of motion of LO state

$$\begin{aligned}&\frac{9}{10}\tilde{\alpha}\varphi(r) - \frac{27}{10}\varphi(r)^3 + \frac{27}{16}\varphi(r)^5 - 2q^2\varphi(r) + q^4\varphi(r) \\ &+ \frac{m^4 - 4m^2}{r^4}\varphi(r) - \frac{2m^2\varphi(r)}{r^2} + \frac{2m^2q^2\varphi(r)}{r^2} + \frac{5}{2}q^2\varphi(r)^3 \\ &+ \frac{9m^2\varphi(r)^3}{2r^2} + \frac{\varphi'(r)}{r^3} + \frac{2m^2\varphi'(r)}{r^3} + \frac{2\varphi'(r)}{r} - \frac{2q^2\varphi'(r)}{r} \\ &- \frac{15\varphi(r)^2\varphi'(r)}{4r} - \frac{15}{4}\varphi(r)\varphi'(r)^2 + 2\varphi''(r) - 2q^2\varphi''(r) \\ &- \frac{\varphi''(r)}{r^2} - \frac{2m^2\varphi''(r)}{r^2} - \frac{15}{4}\varphi(r)^2\varphi''(r) \\ &+ \frac{2\varphi^{(3)}(r)}{r} + \varphi^{(4)}(r) = 0.\end{aligned} \tag{4}$$

There are three parameters $\tilde{\alpha}, m, q$ in the equations. m is the winding number of the vortex. We may call $\tilde{\alpha}$ 'external field' corresponding to the temperature and Zeeman effect, and it goes from 0.8999 to 1.13333 [16]. $q = \frac{2\pi}{Z}$ is the modulation vector along z-direction. We should stress here that it is q that determines the natural period Z in order to minimize the free energy [16].

3.2. The asymptotic behavior at the boundary

We use the Thomas–Fermi approximation to analyze the asymptotic behavior far from the center of the vortex. When $r \gg 1$, we neglect the quantum pressure terms (derivatives with respective to the coordinates r) in Eq. (4) and easily find the physical solution of Eq. (4)

$$\varphi(r) = \left[\frac{4}{5} - \frac{20}{27}q^2 - \frac{4}{3}\frac{m^2}{r^2} + \left(\frac{16}{25} - \frac{32}{729}q^4 - \frac{8}{15}\tilde{\alpha} - \frac{128}{135}\frac{m^2}{r^2} + \frac{64}{81}\frac{m^2q^2}{r^2} + \frac{64}{27}\frac{m^2}{r^4} + \frac{32}{27}\frac{m^4}{r^4}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}. \tag{5}$$

The two parameters $\tilde{\alpha}$ and q determine the convergent value of φ, which decreases when $\tilde{\alpha}$ or q increases. The result of T–F approximation is shown in Fig. 1. From (a) we know that for fixed $\tilde{\alpha}$, the scalar reversely increases with q. The proper q is the middle one which is corresponding to the minimum of $\tilde{\mathcal{F}}$. For one fixed value of $\tilde{\alpha}$, there is only one proper value of q to minimize the free energy $\tilde{\mathcal{F}}$. In (b) we show the different $\tilde{\alpha}$ with their proper q, the scalar decreases with the increasing $\tilde{\alpha}$.

Now we consider the asymptotic behavior of the equation of motion at $r \to 0$. When $r \to 0$, we assume $\varphi(r) = r^\chi$. After substituting it into Eq. (4), we get the leading order term

$$\left(m^4 - 4m^2 + 4m^2\chi + 4\chi^2 - 2m^2\chi^2 - 4\chi^3 + \chi^4\right)r^{\chi-4}. \tag{6}$$

To avoid singularity at $r \to 0$, we find $\chi = |m|$, or $\chi = 2 + |m|$. Thus we get:

$$\varphi(r) = c_1 r^{|m|} + c_2 r^{2+|m|} \quad \text{when } r \to 0. \tag{7}$$

The asymptotic behavior of φ at $r \to 0$ is different from the usual vortices, where $\varphi(r) = c_1 r^{|m|}$. They grow more faster near $r = 0$.