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Electron Sagnac gyroscope in an array of mesoscopic quantum rings

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ABSTRACT

The Sagnac effect is an important phase coherent effect in optical and atom interferometers where rotations with respect to an inertial frame are measured in the interference pattern. We analyze the Sagnac effect in a serial array of mesoscopic ring shaped electron interferometers comprised of rings with half-circumferences comparable to the mean free path. The entire array is, however, much larger than the phase coherence length. Phase coherent transport at the level of individual rings leads to a measurable Sagnac effect in the conductance of the chain. We use the signal to noise ratio (SNR) to determine the number of rings needed to measure a desired rotation rate.

1. Introduction

Matter wave interferometry is a key paradigm of quantum mechanics. Its roots lie at the very foundation of quantum theory as a way of experimentally demonstrating the wave properties of matter. Recently matter wave interferometry with laser cooled atoms has shown great promise for novel sensing devices [1] ranging from precision measurements of inter-atomic forces to gravity gradiometers for geophysical prospecting and tests of general relativity [2,3].

One of the most exciting applications of atom interferometry is the measurement of inertial rotations via the Sagnac effect, which causes the effective path length of two arms of an interferometer to differ by an amount proportional to the rotation rate Ω perpendicular to the plane of the interferometer. In general for a matter wave interferometer (MI) with particles of mass M, the phase shift in the interference fringes induced by Ω is $\Theta_S = 2MA\Omega/\hbar$, where A is the area enclosed by the two interferometer paths [4]. In comparison to an optical interferometer gyroscope of equal A that also operates on the basis of the Sagnac effect [5,6], the phase shift and hence sensitivity to rotations is $Mc^2/\hbar\omega$ larger for the MI than the optical gyroscope [7,8]. (Here ω is the frequency of the light.) For atoms, this represents a 10^{10} enhancement of the phase shift while for electrons the enhancement would be 10^6. Since optical Sagnac gyroscopes have already found wide commercial application with the military and commercial aviation for inertial navigation and positioning and stabilization, the additional potential 10^{10} sensitivity with atoms has been a strong stimulus for atom interferometry research leading to numerous experimental demonstrations of atom interferometer Sagnac gyroscopes with sensitivities surpassing the best commercial optical devices [2,3,9,10].

Since the Sagnac effect applies to all types of MI's, it should *in principle* also be observable with electrons in a phase coherent solid state interferometer such as those used in studies of the Aharonov–Bohm (AB) effect in metals [11], semiconductors [12], and graphene [13]. The Sagnac effect has been observed using an electron interferometer in vacuum [14] and at the same time it is worth pointing out that the Sagnac effect is mathematically identical to the AB effect [15]. However, a quick estimate of the size of Θ_S for an electron interferometer with circumference equal to a typical phase coherence length $\ell_\phi \sim 10$ μm, shows that Θ_S is far too small to be measured for values of Ω that could be produced in a laboratory. This begs the question is there a way to scale up the Sagnac signal to a level that is readily detectable? Here we propose a novel type of solid state device consisting of a serial array of mesoscopic ring shaped interferometers connected via quantum wires. Although the overall device dimensions is orders of magnitude larger than either the mean free path or phase coherence length, phase coherence at the level of individual interferometers still produces a Sagnac effect that can be measured in the conductance of the chain for a sufficiently large number of rings.

Our analysis includes an analytic expression for the conductance of the chain in the presence of Sagnac rotational phase shifts. To obtain this analytic expression we make two key simpli-

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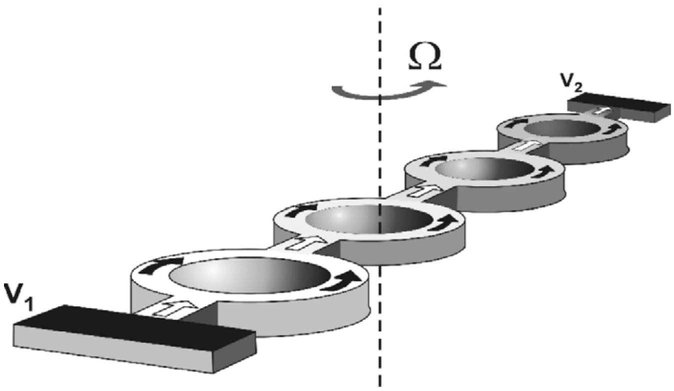

Fig. 1. Schematic diagram of a chain of ring shaped quasi-ballistic electron interferometers connected via nanowires and between leads with bias voltage $\Delta V = V_1 - V_2$.

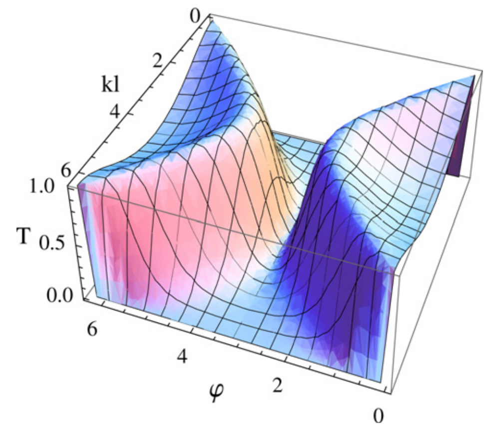

Fig. 2. Transmission coefficient as a function of both kl and φ for a single ring.

fying assumptions. First we assume single mode transport in the rings. Secondly, it is assumed that for a chain of N rings, the electrons propagate through $n < N$ rings without having their phases randomized in a quasi-ballistic manner. These rings serve as the Sagnac interferometers that generate the rotation induced effect on the conductance. The remaining $N - n$ rings along with the interconnects between rings, simply give rise to an overall additional resistance. For rings with half-circumferences less than or equal to the mean free path, $n \approx N$. We calculate the signal to noise ratio (SNR) for the chain by including the effect of Johnson–Nyquist thermal noise and shot noise and show that it is possible to achieve $SNR > 1$ for sub-Hz rotation rates in a chain with an overall size of about 1 cm^2. The assumptions allow us to capture the essential physics in a succinct manner with it to be understood that more precise quantitative predictions required for any experiment require numerical simulations that are beyond the scope of this work.

2. Model

The interferometer chain shown in Fig. 1 consists of an array of conducting micron size rings with a nanowire between each ring and the entire array is situated between two macroscopic leads with bias potential ΔV between them. Here we do not enter into details about the materials in which the device is fabricated but instead characterize the system by a mean free path ℓ_{mfp}, phase coherence length ℓ_ϕ, effective mass m^* of the charge carriers (electrons), and an overall resistance for the nanowires. It is assumed that the device could be fabricated in semiconductors using standard lithographic techniques or in carbon based conductors such as graphene. Here we also assume that $\ell_{mfp} \approx \ell_\phi$.

The half-circumference, ℓ, of the rings is assumed to be less than ℓ_ϕ so that in general electrons entering the ring are split between the two arms and are later recombined at the other end without undergoing any phase decoherence. If a ring rotates at a rate Ω about an axis perpendicular to the plane of the rings, a path difference is created in the two branches of the ring segments $\ell \pm \delta\ell = \ell \pm \Omega A/v$ so that when the electron de Broglie waves recombine, they are out of phase by the amount

$$\Theta_S = 2k\delta\ell = 2\Omega A m^*/\hbar,$$

which is the Sagnac effect for a ring with enclosed area $A = \pi R^2$ and electrons with effective mass m^* [17]. For an array of rings, the axis of rotation does not correspond in general to the center of each ring. However, it can be shown that in the calculation of the Sagnac phase shift, the terms proportional to the distance R_0 from the axis of rotation to the ring center cancel such that the above expression Θ_S continues to be valid [19]. In general we define the relative phase shift between arms of each ring as

$$\varphi_i = \Theta_S^{(i)} + \theta = \frac{2\pi m^* R_i^2 \Omega}{\hbar} + \theta, \tag{1}$$

which takes into account variations in the sizes of rings through the radius $R_i = \ell_i/\pi$. An additional phase shift θ is introduced, which is used to tune the interference between paths to ensure maximum sensitivity of the gyroscope. It can be controlled in various ways including applying an external magnetic field to the device to control the Aharonov–Bohm flux or designing the rings with a path difference that corresponds to the desired phase offset.

Connecting the rings are nanowires, which because of their width, length, and/or bulk mobility, it is assumed that the electron transport is incoherent in these segments and therefore they are fully characterized by their resistivities z_j for $j = 1, \dots, N-1$. The probability that an electron propagates a distance l without having its phase randomized is $P(l) = \exp(-l/\ell_\phi)$ from which one can see that for an ensemble of N rings with half-circumference ℓ, the number n of rings that do contribute a Sagnac phase is $n = N\exp(-\ell/\ell_\phi)$ while for the remaining $\delta N = N - n$ the relative phases of the electrons are randomized yielding no interference. The incoherent contribution of the δN rings can be included in the resistivities of the nanowires.

Using the Landauer–Buttiker formalism, we calculate the conductance of each ring [16]. It has been shown that the treatment of the Aharonov–Bohm (AB) effect and Sagnac effects are mathematically identical for a closed path interferometer [15]. Therefore, we may use the transmission formula for a ballistic ring with electrons of wave number k in the presence of an AB flux [18] and simply substitute φ_i for the AB phase [20],

$$\tilde{T}_i = \frac{64(1-\cos(kl_i))(1+\cos(\varphi_i))}{4[(1+4\cos(\varphi_i)-5\cos(kl_i))^2+(4\sin(kl_i))^2]}. \tag{2}$$

Fig. 2 indicates that for any kl_i there will be a φ_i that maximizes the transmission coefficient. By setting the $\tilde{T}_i$ to its maximum value of 1 we obtain analytical expressions for the values of φ_i at $\tilde{T}_i = 1$,

$$\varphi_i = \pm\cos^{-1}\left(\frac{1+3\cos(kl_i)+4\sqrt{1-\sin^2(kl_i)-\cos^2(kl_i)}}{4}\right). \tag{3}$$

Fig. 2 shows that the choice of kl_i only effects the location where transmission is maximum and can be chosen to conveniently maximize the response to changes in φ. In what follows, we assume that all rings are of the same size with identical R_i and l_i in order to derive an analytic expression for the conductance. Under