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A simple observer design of the generalized Lorenz chaotic systems

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ABSTRACT

In this Letter, the generalized Lorenz chaotic system is considered and the state observation problem of such a system is investigated. Based on the time-domain approach, a simple observer for the generalized Lorenz chaotic system is developed to guarantee the global exponential stability of the resulting error system. Moreover, the guaranteed exponential convergence rate can be correctly estimated. Finally, a numerical example is given to show the effectiveness of the obtained result.

1. Introduction

Recently, several kinds of chaotic systems have been widely investigated; see, for example, [1–26] and the references therein. This is due to theoretical interests as well as to a powerful tool for chaos synchronization and chaos control design. Frequently, chaos in many systems is a source of instability and a source of the generation of oscillation. Chaotic systems commonly exist in various fields of application, such as system identification, secure communication, and ecological systems. As we know, it is either impossible or inappropriate, form practical considerations, to measure all the elements of the state vector. Furthermore, the observer has come to take its pride of place in system identification, control design, and filter theory. Besides, the observer design for the state reconstruction of dynamic systems with chaos is in general not as easy as that without chaos. Based on the above-mentioned reasons, the observer design of chaotic systems is really meaningful and crucial.

In this Letter, the state reconstructor for the generalized Lorenz chaotic system is considered. Using the time-domain approach, an observer for the generalized Lorenz chaotic system is provided to guarantee the global exponential stability of the resulting error system. Meanwhile, the guaranteed exponential convergence rate can be accurately estimated. Finally, a numerical example is provided to illustrate the effectiveness of the obtained result.

2. Problem formulation and main result

In this Letter, we consider the following generalized Lorenz chaotic systems

$$\begin{cases} \dot{x}_1(t) = \left(10 + \dfrac{25}{29}k\right) \cdot \left[x_2(t) - x_1(t)\right], \\ \dot{x}_2(t) = \left(28 - \dfrac{35}{29}k\right)x_1(t) + (k-1)x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = \left(\dfrac{-8}{3} - \dfrac{1}{87}k\right)x_3(t) + x_1(t)x_2(t), \\ y(t) = x_1(t), \end{cases} \tag{1}$$

where $x(t) := [x_1(t)\ x_2(t)\ x_3(t)]^T \in \Re^3$ is the state vector, $y(t) \in \Re$ is the system output, and k is the system parameter with

$$k \in \{a \mid -232 < k < -11.6\} \cup \{a \mid k > -11.6\}.$$

Remark 1. It is noted that the system (1) displays chaotic behavior for each $0 \leqslant k \leqslant 29$ [21]. The Lorenz system, generalized Chen system, and generalized Lü one [21] are the special cases of system (1) with $k = 0$, $k \in \{a \mid 23.2 < k \leqslant 29\}$, and $k = 23.2$, respectively. Generally speaking, the tasks of observer-based control systems (with or without chaos) can be divided into two categories: observer-based stabilization (or regulation) and tracking (or synchronization). In stabilization problems, an observer-based control system, called an observer-based stabilizer, is to be designed so that the state of the closed-loop system will be stabilized. Examples of observer-based stabilization tasks are temperature control

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of refrigerators, position control of robot arms, and suppression of chaos. In tracking control problems, the design objective is to construct an observer-based controller, called an observer-based tracker, so that the system output tracks a given time-varying trajectory. Making a robot hand draw straight lines or circles, making an aircraft fly alone a specified path, synchronization of master-slave chaotic systems, and secure communication are typical tracking control tasks. For more detailed knowledge, one can refer to [27–33].

It is a well-known fact that since states are not always available for direct measurement, states must be estimated. The objective of this Letter is to search an observer for the system (1) such that the global exponential stability of the resulting error systems can be guaranteed. In what follows, $\|x\|$ denotes the Euclidean norm of the column vector x and $|a|$ denotes the absolute value of a real number a.

Before presenting the main result, let us introduce a definition which will be used in the main theorem.

Definition 1. The system (1) is exponentially state reconstructible if there exist an observer $E\dot{\hat{x}}(t) = h(\hat{x}(t), y(t))$ and positive numbers k and a such that

$$\|e(t)\| := \|x(t) - \hat{x}(t)\| \leqslant k\exp(-\alpha t), \quad \forall t \geqslant 0,$$

where $\hat{x}(t)$ expresses the reconstructed state of the system (1). In this case, the positive number α is called the exponential convergence rate.

Now we present the main result for the state observer of system (1).

Theorem 1. *The system (1) is exponentially state reconstructible. Besides, a suitable observer is given by*

$$\begin{cases} \hat{x}_1(t) = y(t), \\ \hat{x}_2(t) = \left(\dfrac{29}{290+25k}\right)\cdot \dot{y}(t) + y(t), \\ \dot{\hat{x}}_3(t) = \left(-\dfrac{8}{3} - \dfrac{1}{87}k\right)\cdot \hat{x}_3(t) + \hat{x}_1(t)\cdot\hat{x}_2(t), \end{cases} \tag{2}$$

with the guaranteed exponential convergence rate $\alpha := \frac{8}{3} + \frac{1}{87}k$.

Proof. From (1), (2) with

$$e_i(t) := x_i(t) - \hat{x}_i(t), \quad \forall i \in \{1,2,3\},$$

it can be readily obtained that

$$e_1(t) = x_1(t) - \hat{x}_1(t) = 0, \quad \forall t \geqslant 0,$$

$$\begin{aligned} e_2(t) &= x_2(t) - \hat{x}_2(t) \\ &= x_2(t) - \left[\frac{29}{290+25k}\dot{y}(t) + y(t)\right] \\ &= x_2(t) - \left[\frac{29}{290+25k}\dot{x}_1(t) + x_1(t)\right] \\ &= x_2(t) - \frac{29}{290+25k}\left[\left(10 + \frac{25k}{29}\right)x_2(t) \right. \\ &\quad \left. - \left(10+\frac{25k}{29}\right)x_1(t)\right] - x_1(t) \\ &= 0, \quad \forall t \geqslant 0, \end{aligned}$$

$$\begin{aligned} \dot{e}_3(t) &= \dot{x}_3(t) - \dot{\hat{x}}_3(t) \\ &= \left(-\frac{8}{3} - \frac{k}{87}\right)x_3(t) + x_1(t)x_2(t) \\ &\quad - \left(-\frac{8}{3} - \frac{k}{87}\right)\hat{x}_3(t) - \hat{x}_1(t)\hat{x}_2(t) \\ &= \left(-\frac{8}{3} - \frac{k}{87}\right)\cdot\left[x_3(t) - \hat{x}_3(t)\right] + x_1(t)\cdot\left[x_2(t) - \hat{x}_2(t)\right] \\ &= \left(-\frac{8}{3} - \frac{k}{87}\right)e_3(t) + x_1(t)\cdot e_2(t) \\ &= \left(-\frac{8}{3} - \frac{k}{87}\right)e_3(t) \\ &= -\alpha e_3(t), \quad \forall t \geqslant 0. \end{aligned}$$

This implies that

$$\begin{aligned} &\frac{d[e_3(t)\exp(\alpha t)]}{dt} = 0 \\ &\Rightarrow e_3(t)\exp(\alpha t) = e_3(0)\exp(\alpha 0) \\ &\Rightarrow e_3(t) = \exp(-\alpha t)e_3(0) \\ &\Rightarrow \left|e_3(t)\right| = \left|e_3(0)\right|\cdot\exp(-\alpha t), \quad \forall t \geqslant 0. \end{aligned}$$

Consequently, we conclude that

$$\|e(t)\| = \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t)} \leqslant \left|e_3(0)\right|\cdot\exp(-\alpha t), \quad \forall t \geqslant 0.$$

This completes the proof. □

3. Illustrative example

Consider the generalized Lorenz chaotic system (1) with $k = 1$. From Theorem 1, we conclude that the system (1) with $k = 1$ is exponentially state reconstructible by the observer

$$\begin{cases} \hat{x}_1(t) = y(t), \\ \dot{\hat{x}}_2(t) = \dfrac{29}{315}\dot{y}(t) + y(t), \\ \dot{\hat{x}}_3(t) = \dfrac{-233}{87}\cdot\hat{x}_3(t) + \hat{x}_1(t)\cdot\hat{x}_2(t), \end{cases}$$

with the guaranteed exponential convergence rate $\alpha = \frac{233}{87}$. The typical state trajectories for the system (1) with $k = 1$, -10, and 50, are depicted in Figs. 1–3, respectively. Furthermore, the time response of error states for the system (1) with $k = 1$, -10, and 50, are depicted in Figs. 4–6, respectively. From the foregoing simulations results, it is seen that the system (1) with $k = 1$, -10, and 50, regardless of chaotic system or nonchaotic system, is exponentially state reconstructible by the observer (2).

4. Conclusions

In this Letter, the generalized Lorenz chaotic system has been considered and the state observation problem of such a system has been investigated. Based on the time-domain approach, a simple state reconstructor for the generalized Lorenz chaotic system has been developed to guarantee the global exponential stability of the resulting error system. In addition, the guaranteed exponential convergence rate can be correctly estimated. Finally, a numerical example has been proposed to show the effectiveness of the obtained result. The observer designs of Rossler system and the specific chaotic DDEs can also be achieved by similar manner. However, the state observer for more complex and higher-dimensional system still remains unanswered. This constitutes an interesting future research problem.