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# New asymptotic stability criteria for neural networks with time-varying delay

Junkang Tian *, Xiangjun Xie

*School of Sciences, Southwest Petroleum University, Chengdu, Sichuan 610500, China*

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ABSTRACT

The problem of delay-dependent asymptotic stability criteria for neural networks with time-varying delay is investigated. A new class of Lyapunov functional is constructed to derive some new delay-dependent stability criteria.The obtained criterion are less conservative because free-weighting matrices method and a convex optimization approach are considered. Finally, numerical examples are given to demonstrate the effectiveness of the proposed method.



## 1. Introduction

In recent years, stability of delayed neural networks have been investigated extensively because of their successful applications in various areas, such as pattern recognition, image processing, association. Thus, the stability of neural networks with time-delay have been widely considered by many researchers [1–24]. The existing stability criteria can be classified into two categories, namely, delay-independent ones [1–3] and delay-dependent ones [4–24]. Generally speaking, the delay-dependent stability criteria are less conservative than delay-independent when the time-delay is small. Therefore, authors always consider the delay-dependent type. Some less conservative stability criteria were proposed in [5] by considering some useful terms and using the free-weighting matrices method. The stability of neural networks with time-varying interval delay were considered in [6] where the relationship between the time-varying delay and its lower and upper bound was taken into account and a less conservative result was derived. Some less conservative criteria of asymptotic stability are derived in [7] by using the free-weighting matrices method and the Jensen integral inequality. But the $\tau(t)-h_1$ and $h_2-\tau(t)$ are all simply enlarged as $h_2-h_1$, it is an important factor in leading to conservatism. An improved delay-dependent stability criterion is derived in [9] by constructing a new Lyapunov functional and using the free-weighting matrices method. However, these results have conservatism to some extent, which exist room for further improvement.

In this Letter, the problem of delay-dependent asymptotic stability criteria for neural networks with time-varying delay is investigated. A new class of Lyapunov functional is constructed to derive some novel delay-dependent stability criteria. The obtained criterion are less conservative because of free-weighting matrices method and a convex optimization approach are considered. Finally, numerical examples are given to indicate significant improvements over the existing results.

## 2. Problem formulation and preliminaries

Consider the following neural networks with time-varying delays

$$\dot{x}(t) = -Cx(t) + Ag\big(x(t)\big) + Bg\big(x\big(t-\tau(t)\big)\big) + \mu \tag{1}$$

* Corresponding author. Tel.: +86 08172642590; fax: +86 08172642360.
*E-mail address:* tianjunkang1980@163.com (J. Tian).

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where $x(t)=[x_1(t),x_2(t),\dots,x_n(t)]^T\in\mathcal{R}^n$ is the neuron state vector, $g(x(\cdot))=[g_1(x_1(\cdot)),g_2(x_2(\cdot)),\dots,g_n(x_n(\cdot))]^T\in\mathcal{R}^n$ denotes the neuron activation function, and $\mu=(\mu_1,\mu_2,\dots,\mu_n)^T\in\mathcal{R}^n$ is a constant input vector. $A,B\in\mathcal{R}^{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively. $C=\mathrm{diag}(C_1,C_2,\dots,C_n)$ with $C_i>0$, $i=1,2,\dots,n$. $\tau(t)$ is time-varying continuous function that satisfies $0\leqslant\tau(t)\leqslant h$, $0\leqslant\dot\tau(t)\leqslant u$, where $h$ and $u$ are constants. In addition, it is assumed that each neuron activation function in (1), $g_i(\cdot), i=1,2,\dots,n$, satisfies the following condition:

$$\gamma_i\leqslant\frac{g_i(x)-g_i(y)}{x-y}\leqslant\sigma_i,\quad\forall x,y\in R,\ x\neq y,\ i=1,2,\dots,n,\tag{2}$$

where $\gamma_i,\sigma_i$, $i=1,2,\dots,n$, are positive constants.

Assuming that $x^*=[x_1^*,x_2^*,\dots,x_n^*]^T$ is the equilibrium point of (1) whose uniqueness has been given in [17] and using the transformation $z(\cdot)=x(\cdot)-x^*$, (1) can be converted to the following system:

$$\dot z(t)=-Cz(t)+Af\big(z(t)\big)+Bf\big(z\big(t-\tau(t)\big)\big)\tag{3}$$

where $z(t)=[z_1(t),z_2(t),\dots,z_n(t)]^T$, $f(z(\cdot))=[f_1(z_1(\cdot)),f_2(z_2(\cdot)),\dots,f_n(z_n(\cdot))]^T$ and $f_i(z_i(\cdot))=g_i(z_i(\cdot)+z_i^*)-g_i(z_i^*)$, $i=1,2,\dots,n$. According to the inequality (2), one can obtain that

$$\gamma_i\leqslant\frac{f_i(z_i(t))}{z_i(t)}\leqslant\sigma_i,\quad f_i(0)=0,\quad i=1,2,\dots,n.\tag{4}$$

In this Letter, we will attempt to formulate some practically computable criteria to check the global asymptotic stability of system (3). The following lemma is useful in deriving the criterion.

**Lemma 1.** *(See [25].) For any constant matrix $\Phi\in\mathcal{R}^{n\times n}$, $\Phi=\Phi^T>0$, scalar $\gamma>0$, vector function $\dot\omega:[0,\gamma]\to\mathcal{R}^n$ such that the integrations concerned are well defined, then*

$$-\gamma\int_{-\gamma}^{0}\dot\omega^T(t+s)\Phi\dot\omega(t+s)\,ds\leqslant-\left(\int_{-\gamma}^{0}\dot\omega(t+s)\,ds\right)^T\Phi\left(\int_{-\gamma}^{0}\dot\omega(t+s)\,ds\right).\tag{5}$$

## 3. Main results

For the asymptotic stability of system described by (3), we have the following result.

**Theorem 1.** *For given scalars $\Gamma=\mathrm{diag}(\gamma_1,\gamma_2,\dots,\gamma_n)$, $\Sigma=\mathrm{diag}(\sigma_1,\sigma_2,\dots,\sigma_n)$, $h\geqslant0$, $u\geqslant0$, the origin of system (3) is globally asymptotically stable if there exist symmetric positive matrices $P$, $Q_1$, $Q_2$, $\left[\begin{smallmatrix}X_{11}&X_{12}\\ *&X_{22}\end{smallmatrix}\right]\left[\begin{smallmatrix}Y_{11}&Y_{12}\\ *&Y_{22}\end{smallmatrix}\right]$, positive diagonal matrices $T_1,T_2$, $\Lambda=\mathrm{diag}(\lambda_1,\lambda_2,\dots,\lambda_n)$, $D=\mathrm{diag}(d_1,d_2,\dots,d_n)$ and any matrices $P_1,P_2,N_i,M_i,L_i,S_i$ $(i=1,2,\dots,7)$ with appropriate dimensions, such that the following LMIs hold*:

$$\mathrm{E}_1=\begin{bmatrix}\bar{\mathrm{E}} & -\frac{h}{2}N\\ * & -\frac{h^2}{4}Q_1\end{bmatrix}<0,\tag{6}$$

$$\mathrm{E}_2=\begin{bmatrix}\bar{\mathrm{E}} & -\frac{h}{2}L\\ * & -\frac{h^2}{4}Q_1\end{bmatrix}<0,\tag{7}$$

$$\Phi_1=\begin{bmatrix}\bar\Phi & -\frac{h}{2}M\\ * & -\frac{h^2}{4}Q_2\end{bmatrix}<0,\tag{8}$$

$$\Phi_2=\begin{bmatrix}\bar\Phi & -\frac{h}{2}S\\ * & -\frac{h^2}{4}Q_2\end{bmatrix}<0,\tag{9}$$

*where*

$$\bar{\mathrm{E}}=[\mathrm{E}_{(i,j)}]\quad(i,j=1,\dots,7),\qquad\bar\Phi=[\Phi_{(i,j)}]\quad(i,j=1,\dots,7),$$
$$\mathrm{E}_{11}=X_{11}+Y_{11}+L_1+L_1^T-P_1C-C^TP_1^T-2\Sigma T_1\Gamma,$$
$$\mathrm{E}_{12}=L_2^T-L_1+N_1,\qquad\mathrm{E}_{13}=X_{12}-L_3^T-N_1,\qquad\mathrm{E}_{14}=L_4^T,$$
$$\mathrm{E}_{15}=P-P_1-C^TP_2^T+L_5^T-\Gamma\Lambda+\Sigma D,$$
$$\mathrm{E}_{16}=Y_{12}+P_1A+L_6^T+T_1(\Gamma+\Sigma),\qquad\mathrm{E}_{17}=P_1B+L_7^T,$$
$$\mathrm{E}_{22}=-(1-u)Y_{11}-L_2^T-L_2+N_2+N_2^T-2\Sigma T_2\Gamma,$$
$$\mathrm{E}_{23}=-L_3^T+N_3^T-N_2,\qquad\mathrm{E}_{24}=-L_4^T+N_4^T,\qquad\mathrm{E}_{25}=-L_5^T+N_5^T,$$
$$\mathrm{E}_{26}=-L_6^T+N_6^T,\qquad\mathrm{E}_{27}=-(1-u)Y_{12}-L_7-N_7^T+T_2(\Gamma+\Sigma),$$