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Full state hybrid lag projective synchronization in chaotic (hyperchaotic) systems ☆

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Abstract

This Letter introduces another novel type of chaos synchronization—full state hybrid lag projective synchronization (FSHLPS), which includes complete synchronization, anti-synchronization, lag synchronization, general projective synchronization and FSHPS in [M. Hu, Z. Xu, R. Zhang, Commun. Nonlinear Sci. Numer. Simul. 13 (2008) 456; M. Hu, Z. Xu, R. Zhang, A. Hu, Phys. Lett. A 361 (2007) 231]. Furthermore, systematic FSHLPS schemes are respectively proposed for the continuous and discrete systems. Finally, some numerical simulations are given to verify the effectiveness of the developed schemes.

Keywords: FSHLPS; Active control; Chaotic (hyperchaotic) systems; Continuous; Discrete

1. Introduction

Since chaos synchronization was first introduced by Pecora and Carroll [1,2], it has been an active research topic in nonlinear science, and has been extensively studied. Over the past 15 years, a variety of approaches have been proposed for chaos synchronization, such as manifold-based method [3], adaptive method [4,5], time delay feedback approach [6], backstepping method [7], nonlinear control scheme [8,9] and many others.

At the same time, many different types of synchronization in chaotic (hyperchaotic) systems were presented. For example, complete synchronization, generalized synchronization, phase synchronization, anti-synchronization, general projective synchronization, lag synchronization and anticipate synchronization, and so on. Recently, Wen [10] presented a full-state projective synchronization between two dynamical systems. For two dynamical systems

$$\dot{x}(t) = F(x)\text{—drive system,}$$

$$\dot{y}(t) = G(x, y)\text{—response system,}$$

where $x = (x_1, x_2, \ldots, x_n)^T$, $y = (y_1, y_2, \ldots, y_n)^T \in R^n$ are state variables of the drive system and the response system, respectively. If there exists a nonzero constant α such that $\lim_{t\to\infty} \|y(t) - \alpha x(t)\| = 0$, $0 \neq \alpha \in \mathbf{R}$, i.e., $\lim_{t\to\infty} |y_i(t) - \alpha x_i(t)| = 0$ $(i = 1, 2, \ldots, n)$, then they call it as full-state projective synchronization (FSPS).

Very recently, Hu [11,12] presented the definition of full-state hybrid projective synchronization (FSHPS). For the above drive system and response system, it is said that they are full state hybrid projection synchronization (FSHPS), if there exists a nonzero

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constant matrix $\alpha = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_n\} \in R^{n\times n}$ such that $\lim_{t\to\infty} \|y(t) - \alpha x(t)\| = 0$, i.e., $\lim_{t\to\infty} |y_i(t) - \alpha_i x_i(t)| = 0$ $(i = 1, 2, \ldots, n)$.

However, in the practical engineering applications, time delay is inevitable. For instance, in the telephone communication system, the voice one hears on the receiver side at time t is often the voice from the transmitter side at time $t - \tau$. So, in many cases, it is more reasonable to require the slave system to synchronize the master system with a time-delay τ. Therefore, motivated by the existing works and take into account of the time-delay, a new kind of chaos synchronization is introduced in this Letter, which is named as full state hybrid lag projective synchronization (FSHLPS). Furthermore, some unified schemes are proposed to realize the FSHLPS. They are respectively illustrated by the continuous systems of hyperchaotic Rössler system and hyperchaotic Lorenz system, also by the discrete systems of the 3D generalized Hénon map and the 3D Baier–Klein map. Meanwhile, numerical simulations are given to verify the effectiveness of the developed methods.

2. Definition of FSHLPS and the unified FSHLPS schemes

First, we will give a new concept of chaos synchronization, which is described as below:

Definition 1 (*FSHLPS*). For the above drive system and response system, it is said that they are full state hybrid lag projection synchronization (FSHLPS), if there exists a nonzero constant matrix $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) \in R^{n\times n}$ such that $\lim_{t\to\infty} \|e(t)\| = 0$, where $e(t) = y(t) - \Lambda x(t-\tau)$, $\tau \geqslant 0$. That is, $\lim_{t\to\infty} |y_i(t) - \lambda_i x_i(t-\tau)| = 0$, $i = 1, 2, \ldots, n$.

Remark 1. Here, we use the term FSHLPS to distinguish the FSPS in Ref. [10] and the FSHPS in Refs. [11,12]. In fact, if $\tau = 0$ and $\lambda_1 = \lambda_2 = \cdots = \lambda_n \neq 0$, it degenerates to be FSPS. And it is simplified to be FSHPS with $\tau = 0$.

Remark 2. It is easy to see that the complete synchronization, anti-synchronization and general projective synchronization are the special cases when scaling matrix Λ equals to I, $-I$ and γI (γ is a nonzero constant and I is the identity matrix) with $\tau = 0$. And it is reduced to be the lag synchronization when $\lambda_1 = \lambda_2 = \cdots = \lambda_n = 1$.

In what follows, a general unified FSHLPS scheme will be proposed for continuous chaotic (hyperchaotic) systems by active control.

Consider a class of n-dimensional chaotic (hyperchaotic) systems in the form of

$$\dot{x}(t) = F\big(x(t)\big), \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in R^n$ is the state vector, and $F(x(t)) = (F_1(x(t)), F_2(x(t)), \ldots, F_n(x(t)))^T \in R^n$ is a continuous nonlinear vector function. We refer to (1) as the drive system and the response system is given by

$$\dot{y}(t) = G\big(y(t)\big) + U(t), \tag{2}$$

where $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$, $G(y(t)) = (G_1(y(t)), G_2(y(t)), \ldots, G_n(y(t)))^T$. And $U(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T \in R^n$ is the controller vector to be determined later.

Let the error vector be $e(t) = y(t) - \Lambda x(t-\tau)$, where Λ is a n-order diagonal matrix, i.e., $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$, $0 \neq \lambda_i \in R$, $i = 1, 2, \ldots, n$. Thus the error dynamical system between the drive system (1) and the response system (2) is

$$\dot{e}(t) = G\big(y(t)\big) - \Lambda F\big(x(t-\tau)\big) + U(t). \tag{3}$$

According to the active control method, choose $U(t) = V(e(t)) + H(x(t-\tau), y(t))$, where $V(e(t)) = (v_1(e(t)), v_2(e(t)), \ldots, v_n(e(t)))^T \in R^n$ is a linear vector function relevant to $e(t)$. Usually, we take $V(e(t)) = Ae(t)$ with A is a $n \times n$ constant matrix.

Theorem 1. *If the suitable vector function $H(x(t-\tau), y(t))$ and matrix A are designed such that all the eigenvalues of error system (3) are on the left-hand side of the complex plane, then the system (1) would be full state hybrid lag projection synchronized with system (2).*

Now, consider the n-dimensional discrete chaotic systems (drive and slave system) which are described by

$$x(k+1) = Ax(k) + F\big(x(k)\big), \tag{4}$$

and

$$y(k+1) = By(k) + G\big(y(k)\big) + U, \tag{5}$$

where $x(k) = (x_1(k), x_2(k), \ldots, x_n(k))^T$, $y(k) = (y_1(k), y_2(k), \ldots, y_n(k))^T \in R^n$ are the state vectors, $F(x(k))$ and $G(y(k))$ are continuous nonlinear vector functions and k is the iteration index. Matrices $A, B \in R^{n\times n}$ and U is the vector controller to be determined for the purpose of making the two different chaotic (hyperchaotic) systems (4) and (5) be in FSHLPS.