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He's variational iteration method for solving a semi-linear inverse parabolic equation

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Abstract

Most scientific problems and physical phenomena occur nonlinearly. Except in a limited number of these problems, we have difficulty in finding their exact analytical solutions. A new analytical method called He's variational iteration method (VIM) is introduced to be applied to solve nonlinear equations. In this work VIM is used for finding the solution of a semi-linear inverse parabolic equation. In this method, general Lagrange multipliers are introduced to construct correction functionals for the problems. The multipliers can be identified optimally via the variational theory. The results are compared with the exact solutions.

Keywords: Inverse problem; Parabolic partial differential equations; Variational iteration method (VIM)

1. Introduction

Most scientific problems and physical phenomena occur nonlinearly. Except in a limited number of these problems, we have difficulty in finding their exact analytical solutions. Therefore, there have been attempts to develop new techniques for obtaining analytical solutions which reasonably approximate the exact solutions [1]. In recent decades, numerical calculation methods were good means of analyzing the nonlinear equations; but as the numerical calculation methods improved, semi-exact analytical methods did, too. Most scientists believe that the combination of numerical and semi-exact analytical methods can also end with useful results. In recent years, several such techniques have drawn special attention, such as Hirtoa's bilinear method [2], the homogeneous balance method [3,4], inverse scattering method [5], the Adomian's decomposition method [6], the homotopy analysis method (HAM) [7], the homotopy perturbation method (HPM) [8–11] and variational iteration method (VIM) [12–15]. In this work the well-known variational iteration method is used for finding the solution of a semi-linear inverse parabolic equation. This method is based on the use of Lagrange multipliers for identification of optimal values of parameters in a functional. Using this method a rapid convergent sequence is produced. The variational iteration method is suitable for finding the approximation of the solution without discretization of the problem [16].

In this Letter we shall consider the following parabolic partial differential equation with a control function in an unbounded domain

$$u_t(x,t) = \Delta u(x,t) + p(t)u(x,t) + \phi(x,t), \quad 0 \leqslant t \leqslant T,\ x \in R^d, \tag{1}$$

with initial condition

$$u(x,0) = f(x), \quad x \in R^d. \tag{2}$$

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And an additional condition as an over specification at a point in the spatial domain in the following form

$$u(x_0,t)=E(t),\quad 0\leqslant t\leqslant T,\ x_0\in R^d,\tag{3}$$

where Δ is the Laplace operator, R^d is the spatial domain of the problem, $d=1,2,3$, $x=(x_1,\ldots,x_d)$, ϕ and E are given while u and p are unknown [16].

Inverse problems of parabolic type arise from various fields of engineering [17–20]. These kind of problems have been investigated by many researchers and recently much attention has been given in the literature to the development, analysis and implementation of accurate methods for the numerical solution of parabolic inverse problems, i.e., the determination of an unknown function $p(t)$ in the parabolic partial differential equations [21–25].

To show the efficiency of the present method, two examples are presented. The results are compared with exact solutions.

2. The application of VIM

To clarify the basic ideas of He's VIM, we consider the following differential equation:

$$Lu+Nu=g(t),\tag{4}$$

where L is a linear operator, N a nonlinear operator and $g(t)$ an inhomogeneous term.

According to VIM, we can write down a correction functional as follows:

$$u_{n+1}(t)=u_n(t)+\int_0^t \lambda\big(Lu_n(\xi)+N\tilde{u}_n(\xi)-g(\xi)\big)\,d\xi,\tag{5}$$

where λ is a general Lagrangian multiplier, which can be identified optimally via the variational theory. The subscript n indicates the nth approximation [12–15].

Before applying this procedure to Eq. (1), following [26,27], we apply a pair of transformations as follows:

$$\omega(x,t)=u(x,t)\exp\left(-\int_0^t p(s)\,ds\right),\tag{6}$$

$$r(t)=\exp\left(-\int_0^t p(s)\,ds\right).\tag{7}$$

Then Eq. (1) transforms to a new non-classic partial differential equation which we call it the reformed equation for (1):

$$\frac{\partial\omega(x,t)}{\partial t}=\Delta\omega(x,t)+r(t)\phi(x,t),\quad 0<t\leqslant T,\ x\in R^d,\tag{8}$$

subject to the initial condition

$$\omega(x,0)=f(x),\quad x\in R^d,\tag{9}$$

and

$$\omega(x_0,t)=r(t)E(t),\quad 0<t\leqslant T.\tag{10}$$

Assume $E(t)\neq 0$, then the later is equivalent to

$$r(t)=\frac{\omega(x_0,t)}{E(t)}.\tag{11}$$

With this transformation, $p(t)$ is disappeared and its role is represented implicitly by $r(t)$. So we overcome the difficulties in handling with $p(t)$ and obtain the following equation:

$$\frac{\partial\omega(x,t)}{\partial t}=\Delta\omega(x,t)+\frac{\omega(x_0,t)}{E(t)}\phi(x,t),\quad 0<t\leqslant T,\ x\in R^d.\tag{12}$$

According to the VIM and Eq. (5), we consider the correction functional in t-direction in the following form [16]:

$$\omega_{n+1}(x,t)=\omega_n(x,t)+\int_0^t \lambda\left(\frac{\partial\omega_n(x,\tau)}{\partial\tau}-\Delta\omega_n(x,\tau)-\frac{\omega_n(x_0,\tau)}{E(\tau)}\phi(x,\tau)\right)d\tau.\tag{13}$$