Physics Letters A 374 (2009) 101–105

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Negative refraction and positive refraction are not Lorentz covariant

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ARTICLE INFO

Article history:
Received 18 August 2009
Received in revised form 15 October 2009
Accepted 16 October 2009
Available online 21 October 2009
Communicated by P.R. Holland

PACS:
03.30.+p
03.50.De
41.20.Jb

ABSTRACT

Refraction into a half-space occupied by a pseudochiral omega material moving at constant velocity was studied by directly implementing the Lorentz transformations of electric and magnetic fields. Numerical studies revealed that negative refraction, negative phase velocity and counterposition are not Lorentz-covariant phenomenons in general.

1. Introduction

The behavior of plane waves at a planar interface between two disparate homogeneous mediums is a central topic in both fundamental and applied electrodynamics. In particular, the phenomenon of negative refraction [1–3] has been the subject of intense research efforts for the past ten years, following experimental reports of this phenomenon in certain metamaterials [4,5]. Much of this effort has been motivated by the development of novel metamaterials, but negative refraction also arises in certain minerals [6] and in biological structures [7]. In addition, the prospects of negative refraction arising in relativistic scenarios — such as in uniformly moving materials [8,9] or in strong gravitational fields [10–13] — is a matter of astrophysical and astronomical significance.

In the following we consider negative refraction induced by uniform motion. Earlier work relating to this topic relied on the Minkowski constitutive relations to describe the moving medium in a nonco-moving inertial reference frame [14–16], per the standard textbook approach [17]. However, this approach is only appropriate to materials which are both spatially and temporally local [18]. More recently, studies based on the uniform motion of realistic materials have been undertaken, using an approach in which the Lorentz transformations of the electric and magnetic fields are directly implemented [8,9]. These studies revealed that the phenomenons of negative phase velocity and counterposition[1] — which are closely allied to negative refraction and similarly associated with certain metamaterials and relativistic scenarios — are not Lorentz covariant. Here we address the hitherto outstanding question: is negative (or positive) refraction Lorentz covariant? By means of a numerical analysis based on a uniformly moving pseudochiral omega material, we demonstrate that the answer to this question is 'no'.

In the notation we adopt, 3-vectors are in boldface with the ˆ symbol denoting a unit vector. Double underlining signifies a 3×3 dyadic (i.e., a second rank Cartesian tensor) and the identity 3×3 dyadic is written as $\underline{\underline{I}} = \hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}} + \hat{\mathbf{z}}\hat{\mathbf{z}}$. The operators Re and Im deliver the real and imaginary parts of complex quantities; and $i=\sqrt{-1}$. The permittivity and permeability of free space are ϵ_0 and μ_0, respectively, with $c_0 = 1/\sqrt{\epsilon_0\mu_0}$ being the speed of light in free space.

2. Refraction into a moving pseudochiral omega material

2.1. Planewave analysis

Our attention is focused on a spatially local, homogeneous material, characterized by the frequency-domain constitutive relations

$$\left.\begin{aligned} \mathbf{D}' &= \underline{\underline{\epsilon}}' \bullet \mathbf{E}' + \underline{\underline{\xi}}' \bullet \mathbf{H}' \\ \mathbf{B}' &= \underline{\underline{\zeta}}' \bullet \mathbf{E}' + \underline{\underline{\mu}}' \bullet \mathbf{H}' \end{aligned}\right\} \tag{1}$$

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[1] See Section 2.2 for definitions of negative refraction, negative phase velocity and counterposition.

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2009.10.042

in the inertial reference frame Σ'. Herein, the 3×3 constitutive dyadics

$$\underline{\underline{\epsilon}}' = \epsilon_0 \begin{pmatrix} \epsilon'_x & 0 & 0 \\ 0 & \epsilon'_y & 0 \\ 0 & 0 & \epsilon'_z \end{pmatrix}, \qquad \underline{\underline{\xi}}' = \frac{1}{c_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -i\xi' & 0 \end{pmatrix},$$
$$\underline{\underline{\zeta}}' = \frac{1}{c_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i\xi' \\ 0 & 0 & 0 \end{pmatrix}, \qquad \underline{\underline{\mu}}' = \mu_0 \begin{pmatrix} \mu'_x & 0 & 0 \\ 0 & \mu'_y & 0 \\ 0 & 0 & \mu'_z \end{pmatrix}. \tag{2}$$

This is a bianisotropic, Lorentz-reciprocal [19] material, known as a *pseudochiral omega material* [20]. Constitutive relations of this form have been used to describe certain negatively refracting metamaterials, assembled from layers of split-ring resonators [21]. Several different designs of metamaterials are based on this general configuration [22–24]. As the pseudochiral omega material is presumed to be dissipative, the constitutive parameters $\epsilon'_{x,y,z}$, ξ' and $\mu'_{x,y,z}$ are complex-valued functions of the angular frequency ω'.

Suppose that the pseudochiral omega material fills the half-space $z>0$, while the half-space $z<0$ is vacuous. The inertial reference frame Σ' translates at constant velocity $\mathbf{v} = v\hat{\mathbf{v}}$ with respect to the inertial reference frame Σ, in the plane of the interface $z=0$. In keeping with an earlier study [9], we take $\hat{\mathbf{v}} = \hat{\mathbf{x}}$. The Lorentz transformations [17]

$$\left.\begin{aligned} \mathbf{E} &= (\mathbf{E}'\bullet\hat{\mathbf{v}})\hat{\mathbf{v}} + \gamma\left[\left(\underline{\underline{I}} - \hat{\mathbf{v}}\hat{\mathbf{v}}\right)\bullet\mathbf{E}' - \mathbf{v}\times\mathbf{B}'\right] \\ \mathbf{B} &= (\mathbf{B}'\bullet\hat{\mathbf{v}})\hat{\mathbf{v}} + \gamma\left[\left(\underline{\underline{I}} - \hat{\mathbf{v}}\hat{\mathbf{v}}\right)\bullet\mathbf{B}' + \frac{\mathbf{v}\times\mathbf{E}'}{c_0^2}\right] \\ \mathbf{H} &= (\mathbf{H}'\bullet\hat{\mathbf{v}})\hat{\mathbf{v}} + \gamma\left[\left(\underline{\underline{I}} - \hat{\mathbf{v}}\hat{\mathbf{v}}\right)\bullet\mathbf{H}' + \mathbf{v}\times\mathbf{D}'\right] \\ \mathbf{D} &= (\mathbf{D}'\bullet\hat{\mathbf{v}})\hat{\mathbf{v}} + \gamma\left[\left(\underline{\underline{I}} - \hat{\mathbf{v}}\hat{\mathbf{v}}\right)\bullet\mathbf{D}' - \frac{\mathbf{v}\times\mathbf{H}'}{c_0^2}\right] \end{aligned}\right\}, \tag{3}$$

with the real-valued scalars

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \qquad \beta = \frac{v}{c_0}, \tag{4}$$

relate the electromagnetic field phasors in the frame Σ to those in the frame Σ'.

Now suppose that the vacuous half-space $z<0$ contains a line source, which is stationary with respect to the frame Σ. The source extends infinitely in directions parallel to the y axis, and it is located at a great distance from the interface $z=0$. Let us consider one plane wave incident on the interface $z=0$, as a representative of the angular spectrum of plane waves launched by the source. With respect to the frame Σ, this plane wave is described by the electric and magnetic field phasors

$$\left.\begin{aligned} \mathbf{E}_i &= \mathbf{e}_i \exp\left[i(\mathbf{k}_i\cdot\mathbf{r} - \omega t)\right] \\ \mathbf{H}_i &= \mathbf{h}_i \exp\left[i(\mathbf{k}_i\cdot\mathbf{r} - \omega t)\right] \end{aligned}\right\}, \quad z \leqslant 0. \tag{5}$$

Herein, the wavevector

$$\mathbf{k}_i = \kappa\hat{\mathbf{x}} + k_0\cos\theta\hat{\mathbf{z}}, \tag{6}$$

with the real-valued scalar

$$\kappa = k_0\sin\theta \in (-k_0, k_0), \tag{7}$$

the free-space wavenumber $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ and ω being the angular frequency with respect to Σ.

With respect to the frame Σ', the incident plane wave is represented by

$$\left.\begin{aligned} \mathbf{E}'_i &= \mathbf{e}'_i \exp\left[i\left(\mathbf{k}'_i\cdot\mathbf{r}' - \omega' t'\right)\right] \\ \mathbf{H}'_i &= \mathbf{h}'_i \exp\left[i\left(\mathbf{k}'_i\cdot\mathbf{r}' - \omega' t'\right)\right] \end{aligned}\right\}, \quad z \leqslant 0, \tag{8}$$

wherein the phasor amplitudes $\{\mathbf{e}'_i, \mathbf{h}'_i\}$ are related to $\{\mathbf{e}_i, \mathbf{h}_i\}$ via the Lorentz transformations (3), while

$$\left.\begin{aligned} \mathbf{k}_i &= \gamma\left(\mathbf{k}'_i\bullet\hat{\mathbf{v}} + \frac{\omega' v}{c_0^2}\right)\hat{\mathbf{v}} + \left(\underline{\underline{I}} - \hat{\mathbf{v}}\hat{\mathbf{v}}\right)\bullet\mathbf{k}'_i \\ \mathbf{r} &= \left[\underline{\underline{I}} + (\gamma-1)\hat{\mathbf{v}}\hat{\mathbf{v}}\right]\bullet\mathbf{r}' + \gamma\mathbf{v}t' \\ \omega &= \gamma\left(\omega' + \mathbf{k}'_i\bullet\mathbf{v}\right) \\ t &= \gamma\left(t' + \frac{\mathbf{v}\bullet\mathbf{r}'}{c_0^2}\right) \end{aligned}\right\}. \tag{9}$$

The incident plane wave gives rise to two refracted plane waves in the half-space $z>0$, and one reflected plane wave in the half-space $z<0$. In the frame Σ', the refracted plane waves are represented by the electric and magnetic phasors

$$\left.\begin{aligned} \mathbf{E}'_t &= \mathbf{e}'_{tj} \exp\left[i\left(\mathbf{k}'_{tj}\cdot\mathbf{r}' - \omega' t'\right)\right] \\ \mathbf{H}'_t &= \mathbf{h}'_{tj} \exp\left[i\left(\mathbf{k}'_{tj}\cdot\mathbf{r}' - \omega' t'\right)\right] \end{aligned}\right\}, \quad z \geqslant 0 \ (j=1,2), \tag{10}$$

wherein the wavevectors

$$\mathbf{k}'_{tj} = \left(\mathbf{k}'_i\bullet\hat{\mathbf{x}}\right)\hat{\mathbf{x}} + k'_{zj}\hat{\mathbf{z}} \quad (j=1,2) \tag{11}$$

comply with Snel's law [17]. The wavevector components k'_{zj}, as well as the relationships between the phasor amplitudes $\mathbf{e}'_{tj}$ and $\mathbf{h}'_{tj}$, are deduced by combining the constitutive relations (1) with the source-free Maxwell curl postulates in Σ' [17]. We find [25]

$$\left.\begin{aligned} k'_{z1} &= \omega'\sqrt{\epsilon_0\mu_0}\sqrt{\mu'_x\left(\epsilon'_y - \frac{(\mathbf{k}'_i\bullet\hat{\mathbf{x}})^2}{\omega'^2\epsilon_0\mu_0\mu'_z}\right)} \\ k'_{z2} &= \omega'\sqrt{\epsilon_0\mu_0}\sqrt{\frac{\epsilon'_x}{\epsilon'_z}\left[\left(\epsilon'_z\mu'_y - \xi'^2\right) - \frac{(\mathbf{k}'_i\bullet\hat{\mathbf{x}})^2}{\omega'^2\epsilon_0\mu_0}\right]} \end{aligned}\right\}. \tag{12}$$

Notice that since k'_{zj} are generally complex-valued, the refracted plane waves are nonuniform.

The reflected plane wave is represented by the electric and magnetic phasors

$$\left.\begin{aligned} \mathbf{E}'_r &= \mathbf{e}'_r \exp\left[i\left(\mathbf{k}'_r\cdot\mathbf{r}' - \omega' t'\right)\right] \\ \mathbf{H}'_r &= \mathbf{h}'_r \exp\left[i\left(\mathbf{k}'_r\cdot\mathbf{r}' - \omega' t'\right)\right] \end{aligned}\right\}, \quad z \leqslant 0, \tag{13}$$

in the frame Σ', with the wavevector of the reflected plane wave being

$$\mathbf{k}'_r = \left(\mathbf{k}'_i\bullet\hat{\mathbf{x}}\right)\hat{\mathbf{x}} - k'_{zr}\hat{\mathbf{z}}. \tag{14}$$

The source-free Maxwell curl postulates in Σ' yield an expression for k'_{zr} and relationships between the phasor amplitudes $\mathbf{e}'_r$ and $\mathbf{h}'_r$.

By invoking the standard boundary conditions across the plane $z=0$, i.e., [17]

$$\left.\begin{aligned} \left(\mathbf{e}'_i + \mathbf{e}'_r\right)\cdot\hat{\mathbf{x}} &= \mathbf{e}'_{tj}\cdot\hat{\mathbf{x}} \quad \left(\mathbf{e}'_i + \mathbf{e}'_r\right)\cdot\hat{\mathbf{y}} = \mathbf{e}'_{tj}\cdot\hat{\mathbf{y}} \\ \left(\mathbf{h}'_i + \mathbf{h}'_r\right)\cdot\hat{\mathbf{x}} &= \mathbf{h}'_{tj}\cdot\hat{\mathbf{x}} \quad \left(\mathbf{h}'_i + \mathbf{h}'_r\right)\cdot\hat{\mathbf{y}} = \mathbf{h}'_{tj}\cdot\hat{\mathbf{y}} \end{aligned}\right\}, \quad z = 0 \ (j=1,2), \tag{15}$$

the phasor amplitudes $\{\mathbf{e}'_r, \mathbf{h}'_r\}$ and $\{\mathbf{e}'_{tj}, \mathbf{h}'_{tj}\}$ can be found. The reflected and the refracted plane waves in the frame Σ may then be deduced by applying the Lorentz transformations (3) and (9).

We represent the refracted plane wave in the frame Σ by the electric and magnetic field phasors

$$\left.\begin{aligned} \mathbf{E}_t &= \mathbf{e}_{tj} \exp\left[i(\mathbf{k}_{tj}\cdot\mathbf{r} - \omega t)\right] \\ \mathbf{H}_t &= \mathbf{h}_{tj} \exp\left[i(\mathbf{k}_{tj}\cdot\mathbf{r} - \omega t)\right] \end{aligned}\right\}, \quad z \geqslant 0 \ (j=1,2), \tag{16}$$

wherein the wavevectors

$$\mathbf{k}_{tj} = \kappa\hat{\mathbf{x}} + k_{zj}\hat{\mathbf{z}} \quad (j=1,2), \tag{17}$$