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Euler–Lagrange equation from nonlocal-in-time kinetic energy of nonconservative system

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ABSTRACT

This Letter focuses on studying generalized Euler–Lagrange equation and Hamiltonian framework from nonlocal-in-time kinetic energy of nonconservative system. According to Suykens' approach, we extend his results and formulate some work related to the nonconservative system. With the Lagrangian and nonconservative force in nonlocal-in-time form, we obtain the higher order generalized Euler–Lagrange equation which leads to an extension of Newton's second law of motion. The Hamiltonian is studied in relation to the Lagrangian in the canonical phase space. Finally, the particle with nonconservative force case is studied and compared with quantum mechanical results. The extended equation gives a possible approach for understanding the connection between classical and quantum mechanics.

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1. Introduction

Nonlocal-in-time is the space–time noncommutative theory. Space–time noncommutative field theories have peculiar properties. Gomis and his partners did a lot of related work in Refs. [1–3], and they got some good results such as the Hamiltonian formalism for space–time noncommutative theories and physical degrees of freedom of nonlocal theories. The nonlocality of finite extent was got by Woodard in Ref. [4]. Llosa [5] gave the Hamiltonian formalism for nonlocal theories.

Apart from these, Feynman [6] noted that the kinetic energy functional can be written as $\frac{1}{2}\frac{(x_{k+1}-x_k)}{\varepsilon}\frac{(x_k-x_{k-1})}{\varepsilon}$ with the position measurements of coordinates x at successive time $t_{i+1}=t_i+\varepsilon$. From Feynman's conclusion, Suykens [7] started from a classical Newtonian mechanics and plugged in a nonlocal-in-time kinetic energy instead of the standard kinetic energy, and he got modification to Newtonian mechanics that could explain the quantum phenomena for the free particle case and harmonic oscillator case, but not in general. In this Letter, we will take Suykens' approach to extend the result from conservative system to nonconservative system and find the difference between these two systems.

As remarked in [7], the kinetic energy and the nonconservative force are obtained in nonlocal form of the nonconservative system. Based on these results, we study the higher order generalized Euler–Lagrange equation which are shown to an extension to the classical Newton's law of motion. The Lagrangian that we gain is singular. Following Suykens [7], in order to connect to the Ostrogradski Hamiltonian [8] of the nonconservative system, we explain the $(1+1)$-dimensional formalism of nonlocal theories [1–3], which has two time coordinates with one local and one nonlocal. It can be considered as a generalization to the Ostrogradski formalism for the case of infinite derivative theories. Compared with quantum mechanics, the particle case with nonconservative force is given to gain further insight into the role of the nonlocality time extent.

This Letter is organized as follows. In Section 2 we study the idea of nonlocal-in-time kinetic energy and nonconservative force. In Section 3 we get the higher order Euler–Lagrange equation for the finite number of derivatives of nonconservative system. In Section 4, the regularization to the singular Lagrangian is made. In Section 5, the singular Lagrangian is connected to the Ostrogradski Hamiltonian. In Section 6 we discuss the Hamiltonian and nonconservative force in the $(1+1)$-dimensional field theory of nonlocal

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theory. In Section 7, the Hamiltonian in nonsingular higher order derivatives form is studied. Finally a simple example will be given.

2. The kinetic energy and nonconservative force in the nonlocal-in-time form

We study the system which subjects to the nonconservative force N. For a nonconservative force, the work done in going from A to B depends on the path taken, such as friction, fluid resistance and air drag. We use a special nonconservative force $N=\dot{q}^2$, which has the same functional form with the kinetic energy. The Hamiltonian action function is defined by $S=\int_{t_1}^{t_2} L\,dt$ where $L=T-V$ denotes the Lagrangian containing the kinetic energy term $T=\frac{1}{2}m\dot{q}^2$ and the potential energy term $V=V(q)$.

Instead of considering the standard form of kinetic energy, we use the generalized coordinates and treat the kinetic energy in nonlocal-in-time form [7] as:

$$T_\varepsilon=\frac{1}{2}m\dot{q}(t)\frac{1}{2}\big[\dot{q}(t+\varepsilon)+\dot{q}(t-\varepsilon)\big]. \tag{1}$$

We take the Taylor approximations

$$q(t+\varepsilon)\approx q(t)+\varepsilon\dot{q}(t)+\frac{\varepsilon^2}{2!}\ddot{q}(t)+\cdots+\frac{\varepsilon^n}{n!}q^{(n)}(t),$$
$$q(t-\varepsilon)\approx q(t)-\varepsilon\dot{q}(t)+\frac{\varepsilon^2}{2!}\ddot{q}(t)+\cdots+(-1)^n\frac{\varepsilon^n}{n!}q^{(n)}(t), \tag{2}$$

where $q^{(n)}(t)$ denote the nth order time-derivatives and ε is a small positive constant and $\varepsilon\ll t$. The interpretation of ε is here only considered at the mathematical level, and Suykens [7] gave the interpretation of the ε at the physical level.

According to [7], one gets the nonlocal-in-time kinetic energy based on the nth Taylor approximations:

$$T_{\varepsilon,n}=\frac{1}{2}m\dot{q}\frac{1}{2}\left[\dot{q}+\sum_{k=1}^{n}\frac{\varepsilon^k}{k!}q^{(k+1)}+\dot{q}+\sum_{k=1}^{n}(-1)^k\frac{\varepsilon^k}{k!}q^{(k+1)}\right]. \tag{3}$$

In this way, the kinetic energy becomes

$$T_{\varepsilon,n}=\frac{1}{2}m\dot{q}^2+\frac{1}{4}m\dot{q}\left[\sum_{k=1}^{n}\big(1+(-1)^k\big)\frac{\varepsilon^k}{k!}q^{(k+1)}\right] \tag{4}$$

and we define a_k as $a_k=1+(-1)^k$, then

$$T_{\varepsilon,n}=T+\frac{1}{4}m\dot{q}\left[\sum_{k=1}^{n}a_k\frac{\varepsilon^k}{k!}q^{(k+1)}\right]. \tag{5}$$

With the special case $n=1$, $T_{\varepsilon,n}=T$, and $n=2$, $T_{\varepsilon,2}=\frac{1}{2}m\dot{q}^2+\frac{1}{4}m\varepsilon^2\dot{q}q^{(3)}$, we denote $L_{\varepsilon,n}=T_{\varepsilon,n}-V$.

Using the same methods, we get the nonconservative force $N=\dot{q}^2$ in the nonlocal-in-time form:

$$N_{\varepsilon,n}=\dot{q}\frac{1}{2}\big[\dot{q}(t+\varepsilon)+\dot{q}(t-\varepsilon)\big] \tag{6}$$

and $\dot{q}$ is the generalized velocity. So we get the nonconservative force based on nth Taylor approximations:

$$N_{\varepsilon-n}=\dot{q}\frac{1}{2}\left[\dot{q}+\sum_{k=1}^{n}\frac{\varepsilon^k}{k!}q^{(k+1)}+\dot{q}+\sum_{k=1}^{n}(-1)^k\frac{\varepsilon^k}{k!}q^{(k+1)}\right]$$
$$=\dot{q}^2+\frac{1}{2}\dot{q}\sum_{k=1}^{n}a_k\frac{\varepsilon^k}{k!}q^{(k+1)}, \tag{7}$$

where a_k were defined before. The nonconservative force is got with the special case $n=1$, $N_{\varepsilon,1}=N$ and $n=2$, $N_{\varepsilon,2}=N+\frac{1}{2}\varepsilon^2\dot{q}q^{(3)}$.

3. Higher order generalized Euler–Lagrange equation of nonconservative system

The Lagrangian $L_{\varepsilon,n}=T_{\varepsilon,n}-V$ contains the higher order derivatives with $L_{\varepsilon,n}(q,\dot{q},\ddot{q},\ldots,q^{(\Upsilon)})$, and the nonconservative force $N_{\varepsilon,n}$ also contains the higher order derivatives with $N_{\varepsilon,n}=N_{\varepsilon,n}(t,q,\dot{q},\ddot{q},\ldots,q^{(\Upsilon)})$, where $\Upsilon=n+1$ denotes the order of the Lagrangian. Note that in relation to a Hamiltonian framework, one can consider the generalized coordinates $q_m(t)$ that $q_m=\dot{q}_{m-1}$ and $m=1,2,\ldots,\Upsilon-1$.

According to [9], the higher order generalized Euler–Lagrange equation of nonconservative system is:

$$\sum_{j=0}^{\Upsilon}(-1)^j\frac{d^j}{dt^j}\frac{\partial L_{\varepsilon,n}}{\partial q^{(j)}}+N_{\varepsilon,n}=0. \tag{8}$$

One has

$$\frac{\partial L_{\varepsilon,n}}{\partial q^{(0)}}=-\frac{\partial V}{\partial q}=F, \tag{9}$$

where F is the conservative force. From Eq. (5), we can get

$$\frac{\partial L_{\varepsilon,n}}{\partial\dot{q}}=m\dot{q}+\frac{1}{4}m\sum_{k=1}^{n}a_k\frac{\varepsilon^k}{k!}q^{(k+1)}, \tag{10}$$

which gives

$$\frac{d}{dt}\frac{\partial L_{\varepsilon,n}}{\partial\dot{q}}=m\ddot{q}+\frac{1}{4}m\sum_{k=1}^{n}a_k\frac{\varepsilon^k}{k!}q^{(k+2)}. \tag{11}$$

For $j\geqslant 2$, we can get

$$\sum_{j=2}^{\Upsilon}(-1)^j\frac{d^j}{dt^j}\frac{\partial L_{\varepsilon,n}}{\partial q^{(j)}}=\sum_{k=1}^{\Upsilon-1}(-1)^{k+1}\frac{d^{k+1}}{dt^{k+1}}\frac{\partial L_{\varepsilon,n}}{\partial q^{(k+1)}}$$
$$=\sum_{k=1}^{\Upsilon-1}(-1)^{k+1}\frac{1}{4}ma_k\frac{\varepsilon^k}{k!}q^{(k+2)}. \tag{12}$$

Together with Eq. (8), we get the following result:

$$N_{\varepsilon,n}+F-m\ddot{q}-\frac{1}{4}m\sum_{k=1}^{n}a_k\frac{\varepsilon^k}{k!}q^{(k+2)}$$
$$+\frac{1}{4}m\sum_{k=1}^{n}(-1)^{k+1}a_k\frac{\varepsilon^k}{k!}q^{(k+2)}=0. \tag{13}$$

From Eq. (7), we can get Eq. (13) as follows:

$$F-m\ddot{q}-m\sum_{k=1}^{n}b_k\frac{\varepsilon^k}{k!}q^{(k+2)}+\dot{q}^2+\frac{1}{2}\dot{q}\left[\sum_{k=1}^{n}a_k\frac{\varepsilon^k}{k!}q^{(k+1)}\right]=0, \tag{14}$$

where $b_k=\frac{1}{4}(1-(-1)^{k+1})a_k$, and $b_k=1$, $a_k=2$ when k is even or $b_k=0$, $a_k=0$ when k is odd.

So for n the generalized Euler–Lagrange equation of nonconservative system is

$$F-m\ddot{q}-m\sum_{k=1}^{n/2}\frac{\varepsilon^{2k}}{(2k)!}q^{(2k+2)}+\dot{q}^2+\dot{q}\left[\sum_{k=1}^{n/2}\frac{\varepsilon^{2k}}{(2k)!}q^{(2k+1)}\right]=0, \tag{15}$$

where only the even derivatives terms remain. For $\varepsilon=0$, we obtain the equation of motion:

$$F-m\ddot{q}+\dot{q}^2=0, \tag{16}$$

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