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Nonlinear adaptive synchronization rule for identification of a large amount of parameters in dynamical models

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ABSTRACT

The existing adaptive synchronization technique based on the stability theory and invariance principle of dynamical systems, though theoretically proved to be valid for parameters identification in specific models, is always showing slow convergence rate and even failed in practice when the number of parameters becomes large. Here, for parameters update, a novel nonlinear adaptive rule is proposed to accelerate the rate. Its feasibility is validated by analytical arguments as well as by specific parameters identification in the Lotka–Volterra model with multiple species. Two adjustable factors in this rule influence the identification accuracy, which means that a proper choice of these factors leads to an optimal performance of this rule. In addition, a feasible method for avoiding the occurrence of the approximate linear dependence among terms with parameters on the synchronized manifold is also proposed.

1. Introduction

Based on intrinsic mechanisms extracted from experimental studies, parameterized dynamical models can be specifically established for various phenomena in many fields, ranging from physics to engineering, from chemistry to biology and even neuroscience [1–4]. The consequence of these mathematical modeling usually invites two questions: "what kind of dynamics corresponding to real phenomena will the models exhibit when a specific group of parameters are taken?" and inversely, "how to accurately and swiftly identify the parameters in the established models when time series are experimentally obtained from real systems?". Researches on the first question usually involve qualitative analysis of dynamical models and present systematic parameter regimes of diverse bifurcations and even chaos. Investigations of the second question have generated enormous literature on the technique of parameters identification in various dynamical models. Representative examples include: least-squares fitting algorithm was developed for parameters identification in the Hodgkin–Huxley model [1]; evolution strategy with a fitness function was utilized to estimate those parameters in the Purkinje Cells model [2,5]; neural networks learning technique was implemented to identify the accurate form of models with or without time delays [6]; extended Kalman filter and the unscented Kalman filter were applied to estimate the reaction rate in biochemical networks [7,8]. Apart from techniques used in parameters identification for models with known vector forms, rational or polynomial L_2 approximation and successive derivatives as embedding coordinates are managed to reconstruct a standard system from scalar observable time series in some typical systems [9,10] and some experimental cases [11] where the vector forms are presumably unknown. However, the aforementioned methods either involve optimization of higher dimensional penalized functions and can be complex to implement in coping with real data, or require several approximations and assumptions. Besides, the reconstructed vector forms of models, relying critically on the implemented observable, are always lack of truly physical or biological meanings [12].

Recently, various types of adaptive techniques based on synchronization between two dynamical systems have been managed

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to realize parameters identification in various nonlinear systems, such as the Lorenz-like systems, several well-known circuit systems, and the neural network systems with or without time delays [13–28]. In comparison with the above-mentioned algorithms, some adaptive techniques based on stability theory of dynamical systems are exceptional: the computation of parameters demands no CPU time or storage intensive arithmetics and may even be adopted as a realtime algorithm [13,14]. Therefore, utilization of adaptive technique to identify parameters in dynamical models is of not only physical significance, but also of practical relevance.

Among all the adaptive techniques, the one, proposed in [13,15,18] and then justified by virtue of the stability theory and invariance principle of dynamical systems in [27], is very convenient for use and efficient. This technique, containing a linear adaptive rule for parameters updating, does not involve the computation of the derivative of the driving signals. Nevertheless, this technique sometimes is not so practical as what was theoretically reported in the literature. For instance, the occurrence of the approximate linear dependence (ALD) among terms with parameters on synchronized manifold always leads to failed identification when the adaptive technique is used [27]. Also, this technique can perform very slowly and even failed identification as the number of parameters becomes large and the length of the sampled time series is finite. Therefore, swift and accurate identification of multiple parameters in a finite time duration is vital to real application, which requires improvement of the existing techniques.

In this Letter, in order to accelerate the convergence rate of parameters identification, we novelly design a nonlinear adaptive rule for parameters updating instead of the conventional linear adaptive rule. The new adaptive rule is verified to be theoretically sound. It also has significant virtues compared to the conventional rule. Interestingly, adjustable factors in the proposed rule can sensitively determine the optimal convergence rate for concrete models, which is illustrated by a representative example in this Letter. We numerically show the robustness of the new rule against various types of noises. Moreover, we propose a feasible method for the avoidance of the above-mentioned ALD. This method, together with the new rule, constitutes a systematic technique to identify a large amount of parameters in real models.

2. Nonlinear adaptive rule

First, consider an n-dimensional dynamical model pending for parameters identification in a general form of

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{a}), \tag{1}$$

where the state variable $\boldsymbol{x}(t) = [x_1(t), \ldots, x_n(t)]^{\mathrm{T}}$ is supposed to be either a continuous driving signal or a discrete time series with a sufficiently small sampling step. Generally, the vector field $\boldsymbol{F}(\boldsymbol{x}, \boldsymbol{a})$, describing the interactions among the state variables, can be modeled by the knowledge from physical, chemical, and even biological theories and experiments. Representative models includes the well-known Chua's circuit [3], the Michaelis–Menten kinetics of enzymatic reactions [4], the Lotka–Volterra models from both ecology and economics [29], and so on. In such models, each component of the vector field in (1) can be expressed particularly by

$$F_i(\boldsymbol{x}, \boldsymbol{a}) = c_i(\boldsymbol{x}) + \sum_{j=1}^{m} a_{ij} f_{ij}(\boldsymbol{x}), \tag{2}$$

where $\boldsymbol{a} = \{a_{ij}\} \in \mathcal{U} \subset \mathbb{R}^n$ are $(n \times m)$ parameters to be identified and $\mathcal{U}$ is some bounded set. $c_i(\boldsymbol{x})$ and $f_{ij}(\boldsymbol{x})$ are assumed to be some real valued functions which are either globally Lipschitz or homogenously polynomial with a degree no more than two with respect to $\boldsymbol{x}$. According to [27] and references therein, the response system with the driving signal $\boldsymbol{x}(t)$ generated by (1) is:

$$\dot{\boldsymbol{y}} = \boldsymbol{F}(\boldsymbol{y}, \boldsymbol{b}) + \boldsymbol{\epsilon} \cdot \boldsymbol{e} + \boldsymbol{\omega} \cdot \boldsymbol{e}^3, \tag{3}$$

where the coupling terms obey the adaptive rules:

$$\dot{\epsilon}_i = -r_i e_i^2, \qquad \dot{\omega}_i = -s_i e_i^4. \tag{4}$$

In particular, the nonlinear adaptive rule for parameters is designed through

$$\dot{b}_{ij} = -\delta_{ij} \cdot g_i(e_i) \cdot f_{ij}(\boldsymbol{y}). \tag{5}$$

Here, the response signal, updated parameters, and updated gain variables can be, respectively, written as $\boldsymbol{y}(t) = [y_1(t), \ldots, y_n(t)]^{\mathrm{T}}$, $\boldsymbol{b} = \{b_{ij}\}$, $\boldsymbol{\epsilon} = [\epsilon_1, \ldots, \epsilon_n]^{\mathrm{T}}$, and $\boldsymbol{\omega} = [\omega_1, \ldots, \omega_n]^{\mathrm{T}}$. The error dynamics $\boldsymbol{e}$ can be componentwise expressed by $e_i = y_i - x_i$, and the initial value for each e_i is taken to be nonzero. Positive constants r_i, s_i and δ_{ij}, regarded as update rates, can be adjusted for optimal performances in synchronization and parameters identification. Each function $g_i(x)$, satisfying $g_i(0) = 0$, is odd, bounded, and monotonically increasing. For example, $g_i(x)$ can be taken as the following sigmoidal function:

$$g_i(x) = \frac{\theta_i}{1 + \mathrm{e}^{-\sigma_i x}} - \frac{\theta_i}{2} \tag{6}$$

with two positive constants σ_i and θ_i. Note that the nonlinear adaptive rule (5) degenerates to the linear rule discussed in [27], when each $g_i(e_i)$ is taken as a linear and unbounded function, i.e. $g_i(e_i) = e_i$. Also note that the degree of the polynomial function in (2) can be greater than two and the vector field can be in a more general form; however, additional coupling terms should be implemented so as to guarantee the rigor of complete synchronization and identification [27,28].

The remainder of this section focuses not only on analytically validating the proposed nonlinear adaptive rule, but also on intuitively elucidating why the proposed rule generically possesses superiority in the convergence rate.

For the first aim, construct the following Lyapunov function candidate:

$$\begin{aligned} &H_{u,M,N}(\boldsymbol{e}, \boldsymbol{\epsilon}, \boldsymbol{\omega}, \boldsymbol{b}) \\ &\quad = \sum_{i=1}^{n} \int_0^{e_i} g_i(s)\,\mathrm{d}s + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} \frac{1}{\delta_{ij}}(b_{ij} - a_{ij})^2 \\ &\qquad + \frac{1}{2}\sum_{i=1}^{n} \frac{u}{r_i}(\epsilon_i + M)^2 + \frac{1}{2}\sum_{i=1}^{n} \frac{u}{s_i}(\omega_i + N)^2, \end{aligned} \tag{7}$$

where u, M and N are some positive constants pending for determination. Generally, the signal $\boldsymbol{x}(t)$ of interest is oscillating or chaotic sampled from nonlinear dynamical systems. Thus, $\boldsymbol{x}(t)$ is reasonably assumed to be bounded, i.e., there exist a series of bounded and closed intervals $\mathcal{A} = \{\mathcal{A}_i\}_{i=1}^n$ such that each component $x_i(t) \in \mathcal{A}_i \subset \mathbb{R}$ for all $t \in [0, +\infty)$. The following arguments follow the main idea of the proofs in [27,28].

Define a metric $D(s, \mathcal{A}_i) = \inf_{q \in \mathcal{A}_i} |s - q|$ for any $s \in \mathbb{R}$ and any interval $\mathcal{A}_i$ derived above. Select arbitrary $\boldsymbol{y}^0 = [y_1^0, \ldots, y_n^0]^{\mathrm{T}} \in \mathbb{R}^n$ and $\boldsymbol{b}^0 = \{b_{ij}^0\} \in \mathbb{R}^{n \times m}$ as the initial values of systems (3) and (5), respectively. Then, set a constant as

$$p \triangleq \sum_{i=1}^{n} \int_0^{D(y_i^0, \mathcal{A}_i)} g_i(s)\,\mathrm{d}s + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} \frac{1}{\delta_{ij}}\left(b_{ij}^0 - a_{ij}\right)^2 \tag{8}$$

and construct a closed ball by