



Soliton interaction in the coupled mixed derivative nonlinear Schrödinger equations

Hai-Qiang Zhang^a, Bo Tian^{a,b,c,*}, Xing Lü^a, He Li^{a,d}, Xiang-Hua Meng^a

^a School of Science, PO Box 122, Beijing University of Posts and Telecommunications, Beijing 100876, China

^b State Key Laboratory of Software Development Environment, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

^c Key Laboratory of Information Photonics and Optical Communications (BUPT), Ministry of Education, PO Box 128, Beijing University of Posts and Telecommunications, Beijing 100876, China

^d Ministry-of-Education Key Laboratory of Fluid Mechanics and National Laboratory for Computational Fluid Dynamics, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

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ABSTRACT

The bright one- and two-soliton solutions of the coupled mixed derivative nonlinear Schrödinger equations in birefringent optical fibers are obtained by using the Hirota's bilinear method. The investigation on the collision dynamics of the bright vector solitons shows that there exists complete or partial energy switching in this coupled model. Such parametric energy exchanges can be effectively controlled and quantitatively measured by analyzing the collision dynamics of the bright vector solitons. The influence of two types of nonlinear coefficient parameters on the energy of each vector soliton, is also discussed. Based on the significant energy transfer between the two components of each vector soliton, it is feasible to exploit the future applications in the design of logical gates, fiber directional couplers and quantum information processors.

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1. Introduction

The propagation of vector optical solitons and their intriguing interaction properties in optical fibers have become a major area of research due to their potential applicability in the optical communication systems, with the coupled nonlinear Schrödinger (CNLS) equations as the focal point [1,2]. Solitons of the CNLS equations consisting of more than one field component are called the vector solitons, which were first suggested for the Kerr nonlinearity [3]. One of the most exciting phenomena associated with solitons is their collisions. It is well known that the collision of scalar Kerr solitons is elastic, i.e., interacting solitons like particles cross each other unaffectedly only by a phase shift, and the physical quantities such as amplitudes and velocities are conserved. This is why scalar Kerr solitons are used as data bits carrier along an optical fiber in optical communication systems [1]. However, for the purpose of information transfer from one soliton to another, scalar Kerr solitons offer very few possibilities. On the contrary, vector solitons are good candidates because they exhibit a host of interesting interacting behaviors; for example, colliding vector solitons can undergo the standard elastic and energy exchange (shape changing) collisions, with partial or complete energy switching between the two components of each vector soliton [4–6].

Recent progress in the study of vector optical solitons and their interaction has paved the way for the development of novel concepts for controlling optical beam diffraction and designing new all-optical devices for light switching, routing and storage [7,8]. Many types of spatial vector optical solitons have been theoretically proposed and experimentally observed in various nonlinear media [9–16]. For instance, in the isotropic Kerr media, the elliptically polarized fundamental vector solitons have been reported by experimental observation

* Corresponding author at: School of Science, PO Box 122, Beijing University of Posts and Telecommunications, Beijing 100876, China.

E-mail address: tian.bupt@yahoo.com.cn (B. Tian).

[9]. The generation of vector trains of dark-soliton pulses in the orthogonal axes of a highly birefringent optical fiber has also been experimentally demonstrated [10]. Moreover, there are many impressive experimental evidence of the ability to exchange energy between the two components of each vector soliton [17–25]. Therefore, it is conceivable that such switching dynamical behaviors can be exploited the future applications in the design of logical gates [17], fiber directional couplers [18] and quantum information processors [26].

In the present study, we examine energy-exchange collisions of vector solitons in the coupled mixed derivative nonlinear Schrödinger (CMDNLS) equations for two polarized components of electric field [27–32]

$$iq_{jt} + q_{jxx} + \mu \left(\sum_{k=1}^2 |q_k|^2 \right) q_j + i\gamma \left[\left(\sum_{k=1}^2 |q_k|^2 \right) q_j \right]_x = 0 \quad (j = 1, 2), \quad (1)$$

where q_j is the slowly varying complex envelope for polarizations, the parameters μ and γ as real constants are respectively the measure of cubic nonlinear strength and derivative cubic nonlinearity. This set of coupled equations is originally derived as a model for describing the pulse propagation in birefringent optical fiber both in picosecond and femtosecond regions [27,28,32]. The case $\gamma = 0$ is the known as the Manakov system [3], and $\mu = 0$ is the coupled derivative nonlinear Schrödinger (CDNLS) equations governing the polarized Alfvén waves in the plasma physics [34]. Therefore, in the presence of two types of nonlinear effects, system (1) could be regarded as a combined CNLS and CDNLS model. In addition, the scalar MDNLS equation ($q_2 = 0$ in system (1)) arises in many different physical applications including the modulated Alfvén wave propagating along a magnetic field in cold plasmas [35], explaining the dynamics of the deformed Heisenberg spin chain [36], the ultrashort pulse propagation in an optical fiber [37,38] and soliton excitations for energy transfer in alpha helical proteins [39]. In Ref. [32], the bilinear equations of system (1) have been presented, but the two-soliton solution obtained from them could satisfy system (1) under the special constraint. This restriction on the parameters will lead to inexistence of the inelastic collision in this coupled model. In this Letter, we will derive the bilinearization of system (1) which is different from that of the same system given in Ref. [32]. In this case, the main importance of the work could result from the bright two-soliton solution of Eq. (1) which shows that collisions with complete or partial switching of energy between vector solitons in the both components can take place in this coupled system.

In the next section, we will apply the Hirota's bilinear method to derive the bright soliton solutions of system (1). In Section 3, through making the asymptotic analysis for the soliton solution, we will examine energy-exchange collisions of vector solitons with partial or complete energy switching. Furthermore, we will carefully analyze the effects of two types of nonlinear parameters on the energy of each vector soliton. Section 4 is allotted for the conclusions.

2. Bright soliton solution

In soliton theory, the Hirota's bilinear method provides a very effective and straightforward procedure to construct the analytic soliton solutions of nonlinear evolution equations [40–43]. Especially, for the integrable evolution equation, the N -soliton solution can be algorithmically obtained by the formal expansion technique.

Through the following gauge transformations

$$q_j = u_j \exp \left[-\frac{i}{2} \gamma \int (|u_1|^2 + |u_2|^2) dx \right] \quad (j = 1, 2), \quad (2)$$

system (1) can be transformed into

$$iu_{jt} + u_{jxx} + \mu \left(\sum_{k=1}^2 |u_k|^2 \right) u_j + i\gamma \left[\left(\sum_{k=1}^2 u_k^* u_{kx} \right) u_j \right] = 0 \quad (j = 1, 2). \quad (3)$$

Using the dependent variable transformations $u_1 = g/f$ and $u_2 = h/f$ where g, h and f are all complex functions and the asterisk denotes the complex conjugate, we can obtain the following bilinear equations of system (3)

$$(iD_t + D_x^2)(g \cdot f) = 0, \quad (iD_t + D_x^2)(h \cdot f) = 0, \quad (4)$$

$$D_x(f \cdot f^*) = \frac{i\gamma}{2} (|g|^2 + |h|^2), \quad (5)$$

$$D_x^2(f \cdot f^*) = \mu (|g|^2 + |h|^2) + \frac{i\gamma}{2} [D_x(g \cdot g^*) + D_x(h \cdot h^*)]. \quad (6)$$

Therefore, by introducing the dependent variable transformations $q_1 = \frac{gf^*}{f^2}$, $q_2 = \frac{hf^*}{f^2}$ and gauge transformations (2), system (1) has the bilinear equations (4)–(6).

The multi-soliton solution of system (1) can be generated by solving the above set of Eqs. (4)–(6) with the power series expansions of g, h and f as

$$g = \varepsilon g_1 + \varepsilon^3 g_3 + \varepsilon^5 g_5 + \cdots, \quad h = \varepsilon h_1 + \varepsilon^3 h_3 + \varepsilon^5 h_5 + \cdots, \quad f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \cdots, \quad (7)$$

where ε is the formal expansion parameter.

2.1. Bright one-soliton solution

In order to obtain one-soliton solutions, we expand the power series (7) to the lowest order

$$g = \varepsilon g_1, \quad h = \varepsilon h_1, \quad f = 1 + \varepsilon^2 f_2. \quad (8)$$

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