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Matter wave interference pattern in the collision of bright solitons

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ABSTRACT

We investigate the dynamics of Bose–Einstein condensates in a quasi one-dimensional regime in a time-dependent trap and show analytically that it is possible to observe matter wave interference patterns in the intra-trap collision of two bright solitons by selectively tuning the trap frequency and scattering length.

1. Introduction

When a gas of massive bosons is cooled to a temperature very close to absolute zero in an external potential, a large fraction of the atoms collapse into the lowest quantum state of the external potential forming a condensate known as a Bose–Einstein condensate (BEC) [1–4]. Bose–Einstein condensation is an exotic quantum phenomenon observed in dilute atomic gases and has made a huge turnaround in the fields of atom optics and condensed matter physics. The experimental realization of BECs in weakly interacting gases [5] has really kickstarted the upsurge in this area of research leading to flurry of activities in this direction while the observation of dark [6] and bright solitons [7], periodic waves [8], vortices and necklaces [9] has given an impetus to the investigation of this singular state of matter.

The dynamics of BECs is governed by an inhomogeneous nonlinear Schrödinger (NLS) equation called the Gross–Pitaevskii (GP) equation and the behaviour of the condensates depends on the scattering length (binary interatomic interaction) and the trapping potential. Eventhough the GP equation is in general nonintegrable, it has been recently investigated for specific choices of scattering lengths and trapping potentials using Darboux [10,11] and gauge transformation approach [12–14].

It is known that a BEC comprises of coherent matter waves analogous to coherent laser pulses and all the atoms are in phase. In other words, the atoms occupy the same volume of space, move at identical speeds, scatter light of the same color and so on. Hence, it looked fundamentally impossible to distinguish them by any measurement. This quantum degeneracy arising out of high degree of coherence has been recently exploited in an experiment by Andrews et al. [15] and Ketterle's group [16] showing that when two separate clouds of BECs overlap under free ballistic expansion, the result is a fringe pattern of alternating constructive and destructive interference just as it occurs with two intersecting laser beams. Javanainen et al. [17] have shown that when two independent condensates are dropped on top of each other, one also observes similar interference pattern with or without phase. In other words, when BECs were made to collide upon release from the trap, de Broglie wave interference pattern containing stripes of high and low density were clearly observed. These experiments which underlined the high degree of spatial coherence of BECs led

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to the creation of atom laser [18]. Can one observe the same matter wave interference pattern by allowing the bright solitons which are condensates themselves to collide in a trap? Motivated by this consideration, we investigate the collisional dynamics of condensates in a time-dependent trapping potential.

2. Gross–Pitaevskii equation and Lax-pair

At the mean field level, the time evolution of macroscopic wavefunction of BECs is governed by the Gross–Pitaevskii (GP) equation,

$$i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}=\left(\frac{-\hbar^2}{2m}\nabla^2+g\left|\Psi(\vec{r},t)\right|^2+V\right)\Psi(\vec{r},t) \tag{1}$$

where $\Psi(\vec{r},t)$ represents the condensate wave function normalized by the particle number $N=\int dr|\Psi|^2$, $g=4\pi\hbar^2 a_s(t)/m$, $V=V_0+V_1$, $V_0(x,y)=m\omega_\perp^2(x^2+y^2)/2$, $V_1=m\omega_0^2(t)z^2/2$. In the above equation, V_0 and V_1 represent atoms in a cylindrical trap and time-dependent trap along z-direction respectively. The time-dependent trap could be either confining or expulsive while the time-dependent atomic scattering length $a_s(t)$ can be either attractive ($a_s<0$) or repulsive ($a_s>0$). As a result, the condensates confront with both time-dependent scattering length and time-dependent trapping potential [19]. Recent experiments have demonstrated that variation of the effective scattering length by even including its sign can be achieved by utilizing the so-called Feshbach resonance [20].

Considering BECs as an assembly of weakly interacting atomic gases and assuming that the transverse confinement is too tight to allow scattering to the excited states of the harmonic trap in the transverse direction under the constraint $a_sN|\Psi|^2\ll 1$ [21,22], we have the following transformation

$$\Psi(\vec{r},t)=\frac{1}{\sqrt{2\pi a_B}a_\perp}\psi\left(\frac{z}{a_\perp},\omega_\perp t\right)\exp\left(-i\omega_\perp t-\frac{x^2+y^2}{2a_\perp^2}\right). \tag{2}$$

Invoking the above transformation, the GP equation reduces to the following form (in dimensionless units)

$$i\frac{\partial\psi}{\partial t}+\frac{1}{2}\frac{\partial^2\psi}{\partial z^2}+\gamma(t)|\psi|^2\psi-\frac{M(t)}{2}z^2\psi=0 \tag{3}$$

where $\gamma(t)=-2a_s(t)/a_B$, $M(t)=\omega_0^2(t)/\omega_\perp^2$, a_B is the Bohr radius, $M(t)$ describes time-dependent harmonic trap which can be either confining ($M(t)>0$) or expulsive ($M(t)<0$). The dynamics of BECs in the presence of a time-independent trapping potential ($M(t)$ is a constant) has already been investigated [10–12]. It should be mentioned that one cannot control the velocity of the solitons when $M(t)$ is a constant. Hence, to generate the bright solitons of Eq. (3) for both regular and expulsive time-dependent potentials, we introduce the following modified lens transformation [23] as

$$\psi(z,t)=\sqrt{A(t)}Q(z,t)\exp\bigl(i\Phi(z,t)\bigr) \tag{4}$$

where the phase has the following simple quadratic form

$$\Phi(z,t)=-\frac{1}{2}c(t)z^2. \tag{5}$$

Substituting the modified lens transformation given by Eq. (4) in Eq. (3), we obtain the modified NLS equation

$$iQ_t+\frac{1}{2}Q_{zz}-ic(t)zQ_z-ic(t)Q+\gamma(t)A(t)|Q|^2Q=0, \tag{6}$$

with

$$M(t)=c'(t)-c(t)^2, \tag{7}$$

and

$$c(t)=-\frac{d}{dt}\ln A(t). \tag{8}$$

Eq. (6) admits the following linear eigenvalue problem

$$\phi_z=U\phi,\qquad U=\begin{pmatrix} i\zeta(t) & Q\\ -Q^* & -i\zeta(t)\end{pmatrix}, \tag{9}$$

$$\phi_t=V\phi,$$

$$V=\begin{pmatrix} -i\zeta(t)^2+ic(t)z\zeta(t)+\frac{i}{2}\gamma(t)A(t)|Q|^2 & (c(t)z-\zeta(t))Q+\frac{i}{2}Q_z\\ -(c(t)z-\zeta(t))Q^*+\frac{i}{2}Q_z^* & i\zeta(t)^2-ic(t)z\zeta(t)-\frac{i}{2}\gamma(t)A(t)|Q|^2\end{pmatrix}. \tag{10}$$

In the above linear eigenvalue problem, the spectral parameter which is complex is nonisospectral obeying the following equation

$$\zeta'(t)=c(t)\zeta(t),\qquad \zeta(t)=\alpha(t)+i\beta(t) \tag{11}$$

with $\gamma(t)=1/A(t)$. It is obvious that the compatibility condition $(\phi_z)_t=(\phi_t)_z$ generates Eq. (6).

Substituting Eq. (8) with $\gamma(t)=1/A(t)$ in Eq. (7), we get

$$\gamma''(t)\gamma(t)-2\gamma'(t)^2-M(t)\gamma(t)^2=0. \tag{12}$$

Thus, the solvability of the GP Eq. (3) depends on the suitable choices of scattering length $\gamma(t)$ and the trap frequency $M(t)$ consistent with Eq. (12). It should be mentioned that the above integrability condition can also be obtained by invoking the following lens transformation $\psi(Z,T)=\frac{1}{\sqrt{\gamma(t)l(t)}}\Phi(Z,T)\exp(ic(t)z^2)$, where $Z=z/l(t)$ and $T=T(t)$ are new independent variables subject to the constrains $c'(t)+2c(t)^2+M(t)/2=0$, $l'(t)-2c(t)l(t)=0$, $T'(t)-l(t)^{-2}=0$ and $c(t)=-\frac{1}{2\gamma(t)}\gamma'(t)$ to convert the GP Eq. (3) into the standard NLS equation, i.e., $i\Phi_T+\frac{1}{2}\Phi_{ZZ}+|\Phi|^2\Phi=0$ [24]. On the other hand, we have transformed Eq. (3) by using the above lens transformation with $l(t)=1$ into modified NLS equation given by Eq. (6) for the same independent variables (z,t) and obtained the integrability condition (Eq. (12)) consistent with the Ref. [24]. We have then generated the Lax-pair of modified NLS equation.

It should also be mentioned at this juncture that the solution of Eq. (12) which originates from the Riccati Eq. (7) is not unique which means that this model is tailor made for realistic experiments. Employing gauge transformation approach [25], one can generate the bright soliton of Eq. (3) as,

$$\psi^1(z,t)=\sqrt{\frac{1}{\gamma(t)}}2\beta_1(t)\,\mathrm{sech}(\theta_1)e^{-\frac{i}{2}c(t)z^2+i\xi_1} \tag{13}$$

where

$$\theta_1=2\beta_1 z-4\int_0^t(\alpha_1\beta_1)\,dt'+2\delta_1,\quad \alpha_1=\alpha_{10}e^{\int_0^t c(t')\,dt'},$$

$$\xi_1=2\alpha_1 z-2\int_0^t\left(\alpha_1^2-\beta_1^2\right)dt'-2\phi_1,\quad \beta_1=\beta_{10}e^{\int_0^t c(t')\,dt'}, \tag{14}$$

where ϕ_1, δ_1, α_{10} and β_{10} are arbitrary real constants. From Eq. (13), one understands that the amplitude of the bright solitons (which are condensates themselves) depends on the scattering length $\gamma(t)$ and the time-dependent trap $M(t)$ ($\beta_1(t)$ is related to $c(t)$ which in turn depends on $M(t)$) while the velocity is governed by the external trap $M(t)$ alone. This gauge transformation approach can be extended to generate multisoliton solutions.