

# Tri-component phononic crystals for underwater anechoic coatings

Honggang Zhao, Yaozong Liu, Jihong Wen, Dianlong Yu, Xisen Wen\*

*Institute of Mechatronical Engineering, and the PBG Research Center, National University of Defense Technology, Changsha 410073, China*

Received 3 November 2006; received in revised form 15 February 2007; accepted 15 February 2007

Available online 20 February 2007

Communicated by R. Wu

## Abstract

Localized resonance in phononic crystal, composed of three-dimensional arrays of composite units, has been discovered recently. The composite unit is a high-density sphere coated by soft silicon rubber. In this Letter, the absorptive properties induced by the localized resonance are systemically investigated. The mode conversions during the Mie scattering of a single coated lead sphere in unbounded epoxy are analyzed by referring the elements of the scattering matrix. Then the anechoic properties of a slab containing a plane of such composite scatterers are investigated with the multiple-scattering method by accounting the effects of the multiple scattering and the viscous dissipation. The results show that the longitudinal to transverse mode conversion nearby the locally resonant region is an effective way to enhance the anechoic performance of the finite slab of phononic crystal. Then, the influences of the viscoelasticity of the silicon rubber and the coating thickness on the acoustic properties of the finite slab are investigated for anechoic optimization. Finally, we synthetically consider the destructive scattering in the finite slab of phononic crystal and the backing, and design an anechoic slab composed of bi-layer coated spheres. The results show that the most of the incident energy is absorbed at the desired frequency band.

© 2007 Elsevier B.V. All rights reserved.

PACS: 43.20.+g; 43.35.+d; 43.40.+s

Keywords: Phononic crystal; Mie scattering; Multiple scattering method; Anechoic material

## 1. Introduction

Recently, there has been growing interest in a special type of inhomogeneous materials, known as phononic crystals (PCs). Within PCs the density and/or elastic coefficients vary periodically in space [1,2]. Based on the Bloch theory, several theoretical methods, such as the plane-wave expansion (PWE) method [1–4], the finite difference time domain (FDTD) method [5–7], the finite element method [8,9] and the multiple scattering (MST) method [10–13], have been developed to analyze the dispersion of the infinite PCs and the propagation of harmonic elastic waves through finite slabs of PCs. Numerous studies of the PCs in one (1D), two (2D), and three dimensions (3D) have been reported in the last few years. Many results of physical interest, for example, the effective homoge-

neous properties, passbands, stopbands (or gaps), localization of classical waves, etc., can be extracted from these studies. On the other hand, it relates to many applications, such as sound/vibration isolation, the quantitative nondestructive evaluation, the design of sound absorptive materials, etc.

More recently, Liu et al. designed a new tri-component PC, i.e., lead spheres coated by soft silicon rubber and arranged periodically in epoxy [14]. They discovered a new low-frequency gap induced by the localized resonance. And an elastic wave incident on such a PC slab would be reflected near the locally resonant range. All these results were got under the condition of fully elasticity. It is known that solid PCs possess several wave polarizations, i.e., one longitudinal and two transverse waves. The mode conversions among these waves and the energy dissipation in viscoelastic PCs, however, have received considerably less attention in Liu's and the following studies. In fact, based on the theory for the monopole or the dipole resonance by isolated spherical inclusions, such as rubber sphere [15], metal sphere [16] or air sphere [17], ideas for the echo reduction

\* Corresponding author. Tel./fax: +86 731 4574975.  
E-mail address: [wenxs@vip.sina.com](mailto:wenxs@vip.sina.com) (X.S. Wen).

have been discussed. More recently, S. Ivansson investigated the sound absorption of the viscoelastic coatings with periodically distributed cavities by accounting for the multiple scattering effects [18,19].

The goal of the present Letter is to examine the viability of using tri-component PCs for anechoic applications. It prefers that the viscoelastic composite inclusions in the rigid matrix can reduce the pressure sensitivity under the practical applications. Firstly, using an exact Mie scattering, we investigate the mode conversion for a coated lead sphere in unbounded epoxy. Then the anechoic properties of a PC slab, containing a plane of coated lead spheres, are investigated by the multiple scattering method by fully accounting for the effects of the multiple scattering and the energy damping of the components. The acoustic properties variation with the rubber’s parameters and the coating thickness are also investigated. Finally, the destructive scattering in the PC slab and the backing are synthetically investigated for the design of practical acoustic absorbers, and a bi-layer PC slab is designed for wide-band anechoic application.

## 2. Models and multiple scattering method

### 2.1. Models

Fig. 1(a) shows the basic arrangement of the anechoic structure including the PC slab under the Cartesian coordinates system. The whole structure can be viewed as a sequence of different layers perpendicular to Z-axis. A plane longitudinal wave is incident from the left half water. For simplifying the analysis, the slab is assumed infinitely along XY plane. The scatterers in XY plane are arranged in a two-dimensional square lattice defined by the primitive vectors  $\mathbf{a}_1$  and  $\mathbf{a}_2$  (see Fig. 1(b)). Fig. 1(c) shows the structure of the unit cell, i.e., the core sphere (with radius  $r$ ) coated by silicon rubber (with radius  $R$ ). The length of the side ( $a$ ) is equal to the lattice constant, i.e., the nearest distance between two neighbor spheres.

### 2.2. Mie scattering

If we assume that the motion is sinusoidal with time dependence of the form  $e^{-i\omega t}$ , the displacement for harmonic elastic

wave propagation in homogeneous elastic medium represents the following time-independent equation

$$(\lambda + 2\mu)\nabla(\nabla \cdot \mathbf{u}) - \mu\nabla \times \nabla \times \mathbf{u} + \rho\omega^2\mathbf{u} = 0. \quad (1)$$

In spherical coordinates system, the solution can be decomposed into one longitudinal (**L** mode) and two transverse (**M** and **N** modes) solutions

$$\mathbf{u} = \mathbf{L} + \mathbf{M} + \mathbf{N}. \quad (2)$$

The process to obtain the Mie scattering matrix ( $T$ ) of a coated sphere is an eigenfunction expansion of the fields in each component. The fields are composed of infinite summations of spherical harmonics with unknown modal coefficients. In the following discussions, the superscripts 1, 2 and 3 will denote the properties of the host, coating layer and core, respectively, [12]. For the host and the coating layer, the displacement fields include both the incident and the scattered waves,

$$\mathbf{u}^1(\mathbf{r}) = \sum_{lm\sigma} a_{lm\sigma} \mathbf{J}_{lm\sigma}^1(\mathbf{r}) + b_{lm\sigma} \mathbf{H}_{lm\sigma}^1(\mathbf{r}), \quad (3)$$

$$\mathbf{u}^2(\mathbf{r}) = \sum_{lm\sigma} c_{lm\sigma} \mathbf{J}_{lm\sigma}^2(\mathbf{r}) + d_{lm\sigma} \mathbf{H}_{lm\sigma}^2(\mathbf{r}), \quad (4)$$

where  $\sigma = 1, 2, 3$  correspond to **L**, **M**, **N** modes,  $\mathbf{J}_{lm\sigma}(\mathbf{r})$  and  $\mathbf{H}_{lm\sigma}(\mathbf{r})$  represent the incident and scattered waves and are defined by the spherical Bessel and Hankel functions of the first kind respectively. The displacement fields in the core sphere with a finite amplitude at the center,

$$\mathbf{u}^3(\mathbf{r}) = \sum_{lm\sigma} e_{lm\sigma} \mathbf{J}_{lm\sigma}^3(\mathbf{r}). \quad (5)$$

Then, the boundary conditions, i.e., the displacement and stress continuity at the inner and outer interfaces of the coating layer, lead to 12 linear equations involving coefficients  $\{a_{lm\sigma}\}$ ,  $\{b_{lm\sigma}\}$ ,  $\{c_{lm\sigma}\}$ ,  $\{d_{lm\sigma}\}$  and  $\{e_{lm\sigma}\}$ . The solution of these equations gives the relations between the coefficients  $B = \{b_{lm\sigma}\}$  and  $A = \{a_{lm\sigma}\}$  with the following scattering matrix form [10],

$$B = TA, \quad (6)$$

where  $T = \{t_{lm\sigma l'm'\sigma'}\}$ . For spherical scatterer, the scattering matrix ( $T$ ) is independent of  $m$  and diagonal with  $l$ . The Mie scattering matrix for each partial waves of  $l$  order presents the

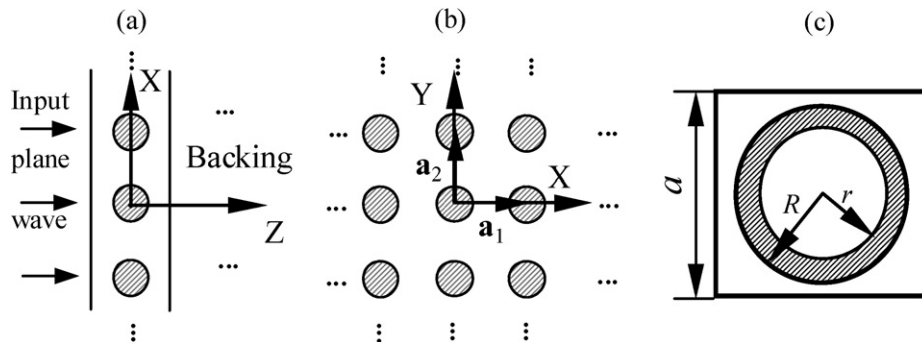


Fig. 1. (a) The anechoic structure includes a backing plate covered by a PC slab. A plane-wave is normally incident from the left along Z-axis. (b) A plane of scatterers with a square lattice, defined by the primitive vectors  $\mathbf{a}_1$ ,  $\mathbf{a}_2$ . (c) The unit cell is composed of a core with radius  $r$  and a coated rubber layer (hatched region) with outer radius  $R$ .

Download English Version:

<https://daneshyari.com/en/article/1862008>

Download Persian Version:

<https://daneshyari.com/article/1862008>

[Daneshyari.com](https://daneshyari.com)