



# The bounds of heavy-tailed return distributions in evolving complex networks



João P. da Cruz<sup>a,b,c</sup>, Pedro G. Lind<sup>b,\*</sup>

<sup>a</sup> Closer, Consultoria, Lda., Av. Eng. Duarte Pacheco, Torre 2, 14<sup>o</sup> C 1070-102 Lisboa, Portugal

<sup>b</sup> Center for Theoretical and Computational Physics, University of Lisbon, Av. Prof. Gama Pinto 2, 1649-003 Lisbon, Portugal

<sup>c</sup> Departamento de Física, Faculdade de Ciências da Universidade de Lisboa, 1649-003 Lisboa, Portugal

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## ABSTRACT

We consider the evolution of scale-free networks according to preferential attachment schemes and show the conditions for which the exponent characterizing the degree distribution is bounded by upper and lower values. Our framework is an agent model, presented in the context of economic networks of trades, which shows the emergence of critical behavior. Starting from a brief discussion about the main features of the evolving network of trades, we show that the logarithmic return distributions have bounded heavy tails, and the corresponding bounding exponent values can be derived. Finally, we discuss these findings in the context of model risk.

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## 1. Introduction: a note on agent models for social systems

Similarly to other fields in social sciences, most of the research made in finance and economics has been dominated by an epistemological approach, in which the behavior of the economic system is explained by a few key characteristics of the behavior itself, like the amplitude of price fluctuations or the analytical form of the heavy-tailed return distributions [1,2]. These key characteristics motivated researchers to assume such distributions as  $\alpha$ -stable Lévy distributions or truncated  $\alpha$ -stable Lévy distributions [3]. The reason for this assumption is given by the more general version of the central limit theorem – sometimes not so well known – which states that the aggregation of a growing number of random variables converges to an  $\alpha$ -stable Lévy distribution [4]. If these random variables have finite variances then the resulting aggregation is a 2-stable Lévy distribution, i.e. a Gaussian distribution. If the variances are infinite – or of the order of the system size – then  $\alpha < 2$  and the so-called heavy-tailed shape emerges as a result of the aggregation. Further, non-Gaussian (heavy-tailed) distributions are associated with correlated variables and therefore it is reasonable to assume that measurements on aggregates of human activities will result in an  $\alpha$ -stable Lévy distribution, since humans are strongly correlated with each other. Henceforth, we refer to  $\alpha$ -stable Lévy distributions with  $\alpha < 2$  as Lévy distributions and with  $\alpha = 2$  as Gaussian distributions.

Without leaving an epistemological approach, we could address the study of the resulting distributions by ignoring the previous arguments and construct a function that fits any set of empirical data just by building up fitting parameters until the plotted function fit the empirical data. Such approach would be the best one, if economic processes were stationary. Unfortunately they are not [5–7] and this means that we cannot disregard the underlying mechanisms generating the data we are analyzing.

Since heavy tails are observed in the returns of economic variables, one would expect that practitioners use Lévy distributions. The particular case of Gaussian distribution was the first to be considered for modeling price of European options, through the well-known Black–Scholes model [8] proposed in 1973. This model ended a story started already in 1900 with Bachelier and his *Theory of Speculation* [9] where Brownian motion was used to model stock price evolution. The Black–Scholes model for option-pricing is however inconsistent with options data, since stock-price behavior is essentially not Gaussian. To overcome the imperfections of the Black–Scholes model, more sophisticated models were proposed since 1980s and 1990s, which basically assume processes more general than Brownian processes. These processes are called Lévy processes [10] and the probability distributions of their increments are infinitely divisible, i.e. one random variable following that probability distribution can be decomposed into one sum of an arbitrary integer number of independent identically distributed random variables.

Still, despite considerable progresses on modeling financial data with Lévy processes, practitioners continue to show a strong preference for the particular class of finite moment's distributions

\* Corresponding author. Tel.: +351 21 790 4862; fax: +351 21 795 4288.  
E-mail address: plind@ci.f.c.ul.pt (P.G. Lind).

and there are good reasons for that. Assuming that Lévy distributions are good representations of economic variables fluctuations, a model based on them is closed when one fits the distribution to empirical data choosing properly the parameter values, which represent the valuable information for financial insight and decision making. However, as said above, fitting is no good when the series are not stationary: there is no guarantee that today's fitting will be the same as tomorrows. Since working with a Gaussian curve is more straightforward than working with a Lévy distribution and needs less parameters for curve fitting, there is no practical gain in abandoning Gaussian distribution to model the distribution of fluctuations according to a prescribed mathematical model, even though it is not entirely correct. In other words, if a Lévy distribution is fitted to empirical data of a non-stationary process one will carry basically the same *model risk*, as if a Gaussian distribution is used.

On a more ontological approach, when modeling financial and economic networks, random variables are translated into agents. Agent-based models for describing and addressing the evolution of markets has become an issue of increasing interest [11] and appeals for further developments [12–15]. They enable one to access three important questions [5]. First, the system is able in this way to be decomposed into sellers and buyers, a common feature of all finance systems. Second, one enables non-stationary regimes to occur, as in real stock markets. Third, by properly incorporating the ingredients of financial agents and the trades among them one can directly investigate the impact of trades in the price, according to some prescribe scheme.

In this Letter we use an agent model for the individual behavior of single financial agents, at a microscopic scale, in a way that the collective behavior generates an output in accordance with the observed curves of macroscopic variables, namely the financial indices. Several of such bottom-up approaches were thoroughly investigated [16,12]. The Solomon–Lévy model [17] defines each agent as a wealth function  $\omega_i(t)$  that cannot go below a floor level, given by  $\omega_i(t) \geq \omega_0 \bar{\omega}(t)$  where  $\bar{\omega}(t)$  is the agent average  $\omega$  at instant  $t$  and  $\omega_0$  is a proper constant. The imposition of the floor based on the mean field  $\bar{\omega}(t)$  means that on average  $\langle |\omega_i(t) - \bar{\omega}(t)| \rangle \sim N$  and, by basic statistics,  $\text{var}(\omega(t)) \sim N^2$ . Consequently, the result of the Solomon–Lévy model, despite the interesting idea of the introduction of a floor similar to what was done by Merton [18] in the agent dynamics, will surely be an  $\alpha$ -stable distribution with a power-law heavy tail, i.e.  $\alpha < 2$ . Percolation based models like Cont and Bouchaud [19] or Solomon and Weisbuch [20], by the nature of the phenomena, also brings up variations of the order of the system size, leading also to Lévy-type distributions.

In our approach, we follow the above considerations, to address the following question: what are the fundamental assumptions, common e.g. to all economic systems, that naturally lead to the emergence of macroscopic distributions that are characterized by heavy tails? Taking an economic system as a prototypical example for the emergence of heavy-tailed distributions, we argue that there are three fundamental assumptions.

First, agents tend to trade, i.e. to interact. Human beings are more efficient in doing specialized labor than being self-sufficient and for that they need to exchange labor. The usage of the expression ‘labor’ can be regarded as excessive by economists, but we look at it as the fundamental quantity that is common to labor, money or wage. Something must be common to all these quantities; if not, we wouldn’t exchange them. The physicists can regard such fundamental quantity as an ‘economic energy’.

Second, we only consume and produce a finite amount of the overall product that exists within our environment. This assumption justifies the emergence for each agent of a maximum production and minimum consumption. If an agent transposes

that finite amount he should not be able to consume anymore.

Third, human agents are different and attract differently other agents to trade. For choosing the way “how” agents attract each other for trading, we notice that this heterogeneity should reflect some imitation, where agents tend to prefer to consume (resp. produce) from (resp. to) the agents with the largest number of consumers (resp. producers). The number of producer and consumer neighbors reflects, respectively, supply and demand of its labor. With such observation it is reasonable to assume that combining both kinds of neighbors should suffice to quantify the price of the labor exchanged.

Heavy-tailed distributions have been subject to intensive research activity till very recently, e.g. when addressing the formation and construction of efficient reservoir networks [21], which shows self-organized criticality with critical exponents that can be explained by a self-organized-criticality-type model. In this Letter, we deal with heavy tails found in economic systems and show that heavy-tailed return distributions are due to the economic organization emerging in a complex economic network of trades among agents governed under the above three assumptions. Further, the model reproducing empirical data is also of the self-organized-criticality-type model, but its main ingredients result from economical reasoning and assumptions.

Our central result deals in particular with the return distribution found in both data and model: we show that the power-law tails are characterized by an exponent that can be measured and is constrained by upper and lower bounds, which can be analytically deduced. The knowledge of such boundaries is of great importance for risk estimates: by deriving upper and lower bounds, one avoids either underestimates, which enable the occurrence of crisis unexpectedly, as well as overestimates, which prevent profit maximization of the trading agents.

We start in Section 2 by describing the ubiquity of heavy tails in financial time series, namely in stock indices. We will argue that such heavy tails result from the combination of a dynamical critical state in real economic systems and an underlying scale-free topology. Applied to a real system such as the financial market, such bounded behavior leads naturally to a maximum and minimum value on risk evaluation, improving the knowledge about the uncertainty of the market future evolution. These bounding values will be derived in Section 3 based in the assumptions listed above and an application to risk model is discussed. Section 4 concludes this Letter.

## 2. Critical behavior underlying return distributions

Heavy tails are observed in return distributions of data in finance and economics. Fig. 1 presents data from several stock market indices. Fig. 1(a) shows the probability density functions (PDF) of the logarithmic returns of each index, symbolized as  $x$ , where one can observe the heavy tails. The exponent characterizing the tails of these distributions are given in Fig. 1(b).

While the heavy-tailed shape of the return distributions was already known and several times reported [22], the explanation for their emergence, and in particular the values of the exponent characterizing them, was up to our knowledge not so frequently addressed.

The emergence of the heavy tails of the return distribution was recently reproduced with a simple model [7] which takes one economic connection as an exchange of labor between two agents, say  $i$  and  $j$ , dissipating an amount of energy  $U_{ij}$ , representing the deficit of  $i$  that results from the labor exchange between  $i$  and  $j$ . Agent  $i$  delivers an amount of labor  $W_{ij}$  to agent  $j$  and gets a proportional amount of “reward”  $E_{ij} = \alpha_{ij} W_{ij}$  where  $\alpha_{ij}$  can be interpreted as an ‘exchange rate’ of labor. Fig. 2 illustrates the eco-

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